

Mathematical investigation: Chessboard Challenge

Learning outcomes in focus

Students should be able to:

U.4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations

U.7 make sense of a given problem, and if necessary, mathematise a situation

U.8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions

U.9 interpret their solution to a problem in terms of the original question

U.11 generate general mathematical statements or conjectures based on specific instances

U.13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely

AF.1 investigate patterns and relationships (linear, quadratic, doubling and tripling) in number, spatial patterns and real-world phenomena involving change

AF.2 investigate situations in which letters stand for quantities that are variable

AF.3 apply the properties of arithmetic operations and factorisation to generate equivalent expressions

Learning intentions

We are learning to:

- break problems down into parts
- investigate and extend a pattern in a systematic way
- represent pattern in tables, charts and graphs
- look for patterns and make conjectures
- validate our conjectures
- generalise our observations
- use words, tables, graphs and letters to communicate and justify mathematical ideas clearly

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Teaching and learning context

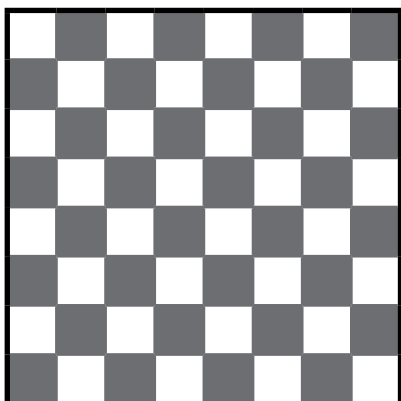
First year students were in the middle of a unit of learning which focused on investigating patterns in a systematic way. This task was used to introduce quadratic sequences. Their task required them to calculate the number of squares on a chessboard. Students recorded their results in tables which helped them spot patterns. They were reminded to record their findings clearly, to make and validate conjectures, to look for patterns and to try to generalise patterns they observed. To start the lesson, students used placemats to brainstorm how they might tackle the problem. We then discussed some strategies as a class. The students worked in groups to solve the problem during a 40 minute class and worked independently to create their speech for homework.

Task

You are a member of the English Debate team and your team is opposed to the following motion.

'There are 64 squares on a chessboard.'

Prepare a speech where you convince your classmates that this statement is wrong. Use tables, charts or graphs to support your arguments. Ensure that you explain clearly how many squares you think there are on a chessboard, and how you arrived at this conclusion.



Extension

Can you find a formula for the number of squares on an $n \times n$ grid.

Success Criteria

I can

SC1 extend a given pattern in a systematic way

SC2 record my observations in a table

SC3 make and verify conjectures

SC4 translate between different representations of a pattern

SC5 draw conclusions from my mathematical ideas

SC6 communicate my ideas clearly in words, tables, graphs or generalised expressions

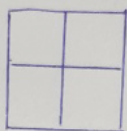
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Rough Work

So each square would be 1x1 so let's try 2x2 and 3x3 to make a bigger square.

Size	No.
1x1	4
2x2	1

T=5

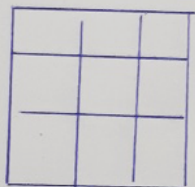


Immediately after the rough work I

realised a pattern

Size	No.
1x1	9
2x2	4
3x3	1

T=14



The numbers are going up in square numbers

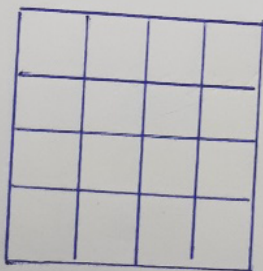
Up til 64

as the board only has 64

squares

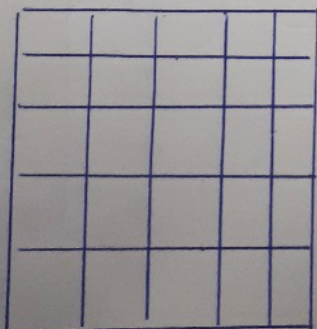
Size	No.
1x1	16
2x2	9
3x3	4
4x4	1

T=30



Size	No.
1x1	25
2x2	16
3x3	9
4x4	4
5x5	1

T=55



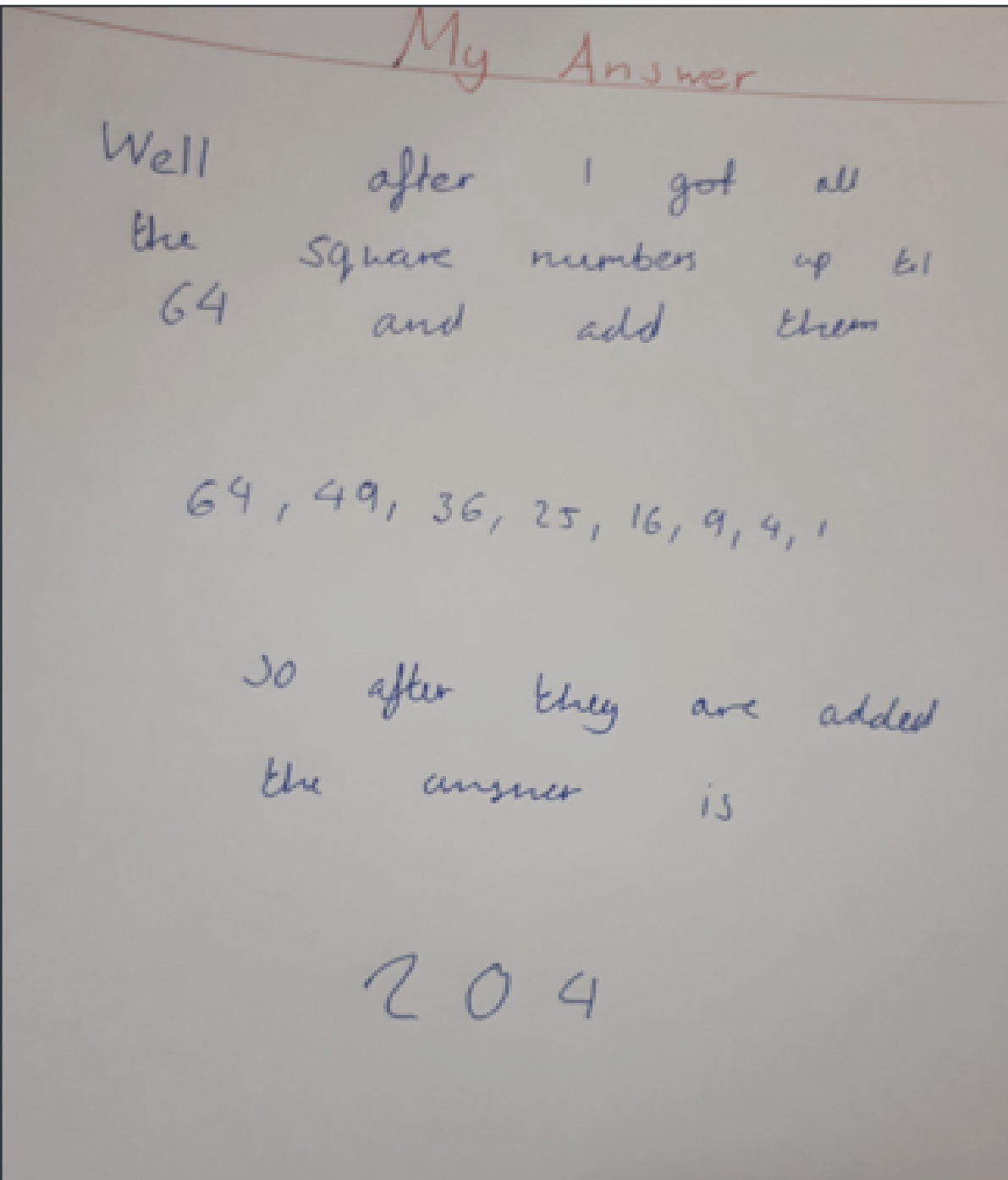
SC3: Makes a conjecture

SC1: Extends a pattern in a systematic way

SC2: Records observations in a table

SC6: Communicates ideas clearly in words and tables

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SC3: Verifies conjecture

SC5: Draws conclusions from mathematical ideas

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Mathematical Inquiry
Chess board

Challenge: My challenge is to calculate the amount of squares in a chessboard

Prediction: I think there will be about 200 squares

Method: I realised each small square is a 1×1 square so then I tried 2×2 , 3×3 and 4×4 . Then I saw a pattern. On an 8×8 square there are:

Size	No.
1×1	64
2×2	49
3×3	36
4×4	25
5×5	16
6×6	9
7×7	4
8×8	1

So if we add these up we could get the answer

} These are all square numbers because the chessboard

SC3: Makes a conjecture

SC1: Extends a pattern in a systematic way

SC2: Records observations in a table

SC6: Communicates ideas clearly in words and tables

Overall judgement:  In line with expectations