Post-Primary

Assessing Mathematics in Post-primary school Junior Cycle

NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

Reviewing Statistics: Supported material for independent study

Throughout your study of statistics and probability you will have considered all aspects of a statistical approach:

- asking a question that results in data that varies
- displaying this data in a way that allows you to see patterns in the variation
- analysing the patterns in the data
- drawing conclusions from that data.

You may even have had an opportunity to get a glimpse of what it is like to become a statistical detective; attempting to account for unexpected variability you observe in a particular set of data.

As you review for the final examination in June, it is important that you can connect each element of your study and consider the BIG IDEA of the strand so that you will be able to use the elements appropriately to help you solve problems that you may not have seen before.

You will complete a statistical investigation as the requirements for CBA 2 in third year.

You may decide to form a study group with your friends or you may prefer to work alone; either way as you work through this review document you will consider issues such as framing a question in order to obtain meaningful ***reliable*** data, selecting a sample in order to avoid ***bias, displaying*** your data in a way that will allow you to see patterns in the variation and ***drawing conclusions*** from your data.

Asking the Question

Think 

Do you use a computer?

How did you answer the question?
What were you thinking when you answered it?

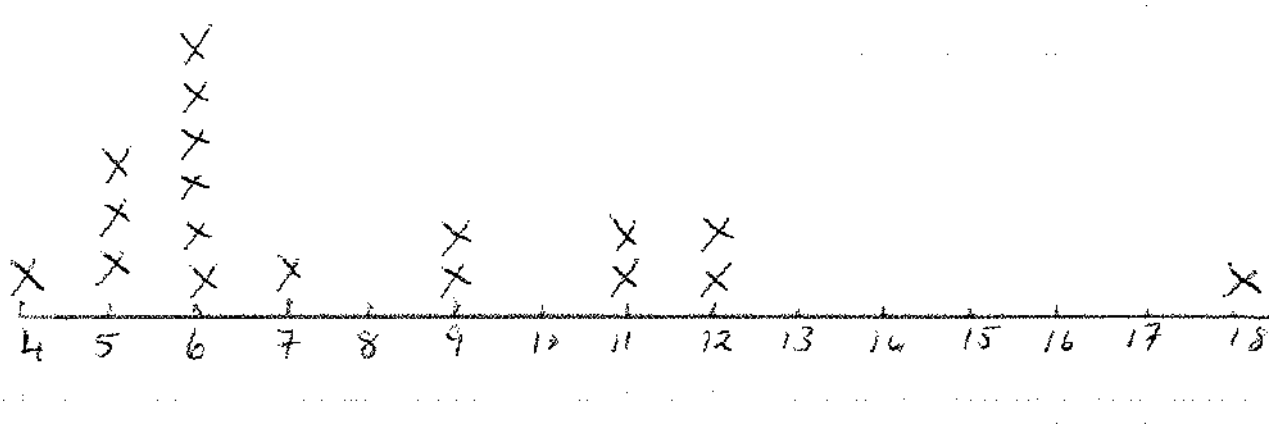
A university sports outlet was considering shutting down their campus shop and becoming an on-line store in an effort to reduce costs. A group of students was surveyed and asked that same question:

Do you use a computer?

Sophie answered **Yes** because she thought the question meant had she ever used a computer.
Joe answered **No** because he thought the question was asking whether he used one regularly.
Andrew answered **No** because he played games on the computer and didn't think this counted as "using" one.
Do you think the results of this survey are ***reliable?***

How could you rephrase the question so that it is less ambiguous and more likely to provide useful answers?

A group of students interested in finding the typical family size for their class obtained the data displayed in this line plot.



What question do you think they asked in order to elicit this data?

What issues would they have needed to consider when framing the question?

Displaying the data and drawing conclusions from it

Use fractions or percentages to describe the data.

Can you see any clumps or areas where a large proportion of the data falls?

Are there any unusual family sizes? [18 is an unusual value in this set.]

What do you think is the typical family size of this group? Why?

If you were asked to predict the family size of someone from this group what value would you give? Why?

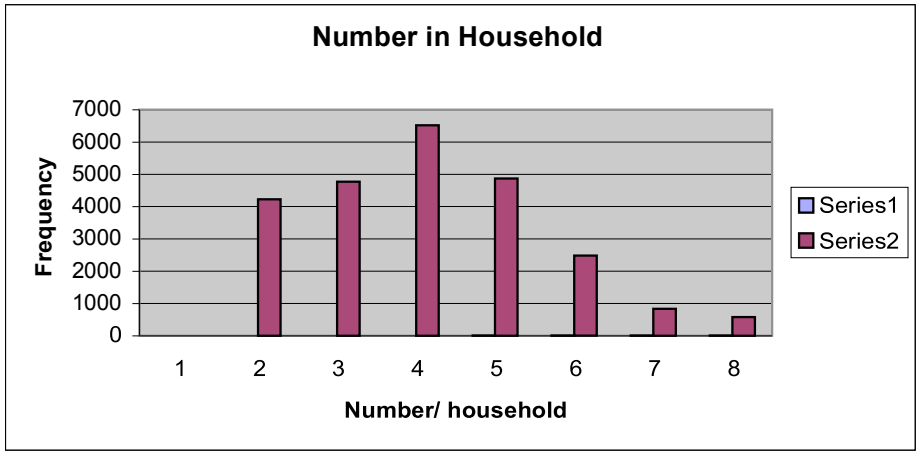
How certain would you be? Can you lower this to a smaller range? How **confident** would you be now?

Calculate the mean family size for this group and identify the median family size. Which is a more reasonable estimate of typicality?

You could do a similar survey of your class, display the data in a line plot and compare the two data sets.

Or you could visit http://beyond2020.cso.ie/Census/TableViewer/tableView.aspx?ReportId=109241 and retrieve some data from the area in which you live, use Excel to display the data and compare it to the sample above.

Household size	Frequency
2	4218
3	4773
4	6512
5	4870
6	2478
7	833
8	572



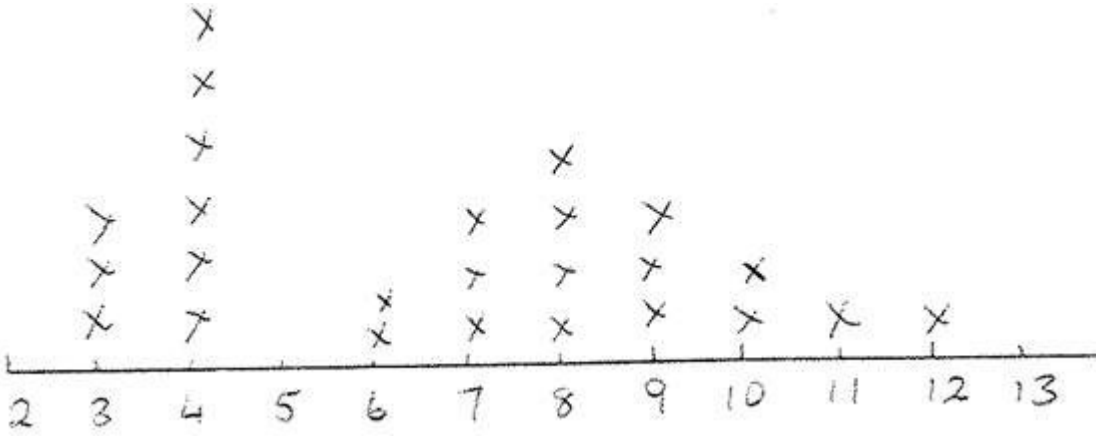
This bar chart was drawn with data from Carlow, compare this data with that from the sample set above.

What is the range of this data set?

What is the range from the sample data set?

Is there any evidence to suggest that the sample was from Carlow? Explain.

The following data set was gathered from a TY class who were interested in finding out what was the typical amount of money spent by their parents on the lotto each week.



Use fractions or percentages to describe the data.

Can you see any clumps or areas where a large proportion of the data falls?

Are there any unusual amounts?

What do you think is the typical amount spent on the Lotto each week by this group? Why?

If you were asked to predict the amount spent on the Lotto each week by someone from this group what value would you give? Why?

How certain would you be? Can you lower this to a smaller range? How **confident** would you be now?

Return to the value you think is the typical amount spent on the Lotto each week by this group. Calculate the mean amount spent on the Lotto by this group and identify the median amount spent on the Lotto each week. Which is a more reasonable estimate of typicality?

The data below was obtained by students trying to find out how good they are at judging a minute.

18	25	26	30	40	41
45	45	47	52	52	56
67	68	74	79	109	

Think 

Will a line plot be a meaningful display for this data? Try it out and see?
Now display the data in a **stem and leaf plot.** Which display is more meaningful? Why?

What is the range of the data? Where is all the data concentrated? Calculate the percentage of data in this region. How good were people at guessing a minute?
Predict how long do you think people in this group think a minute is? How **confident** are you of this answer?

Stop and think

Under what conditions would a **line plot** be a meaningful representation?

Under what conditions would a **stem and leaf plot** be a more meaningful representation?

Try using statistics to solve this problem.

PROBLEM: Climbing helmets are made in a variety of styles and sizes.
The manager of You Climb Safely must decide what styles of helmet to keep in stock and how many helmets of each size to order. A standard fit helmet is offered in 10 sizes. When you order helmets you must order 1000. How many of each helmet size should the manager order?

In order to get an idea of how head sizes are **distributed** the manager decided to measure the head circumferences of a group of people.

Think: what is the **population** of interest? Can he measure the circumferences of the heads of the whole population? How will he choose a **sample**?

The manager chose a **Simple Random Sample** of climbers from clubs around the country and recorded their head circumference and gender ion the table overleaf.

Is this a suitable sample? Why or Why not?

Gender	Head Circumference (mm)
F	522
M	580
M	552
F	531
M	563
F	546
F	545
M	545
M	545
M	568
F	560
M	613
F	555
F	573
M	585
F	584
M	600
M	595
M	593
F	590
M	594
F	564
F	536
M	586
F	540
M	585
M	550
M	565
F	600
F	590
F	551
M	590
M	580
F	577

Is a line plot a good representation of this data?

Display the data in a stem and leaf plot.

Describe the data…..Are there any clumps or areas where the data is concentrated? Are some head sizes more common than others?

Use your representation to answer the original question: ***how many helmets of each size should the manager order?***

Begin by counting the number of leaves on each stem.

Look at the first stem…52 ..How many leaves are there on stem 52? What fraction of the total is this? What % of the total number of head circumference measurements does stem 52 represent?

How many helmets size 520cm- 530cm should the manager order?

Continue working like this until you have decided how many helmets of each size the manager should order.

Return to your representation…Do you think there are **gender effects? That means do you think there is any difference between the data for men and women?** Try representing the male and female data in ***back to back stem plots*** Compare the two sets of data; is there any evidence to suggest that there are differences in the sizes of heads of men and women?

If there are gender effects will this affect the number of helmets the manager should order? Or are helmets unisex?

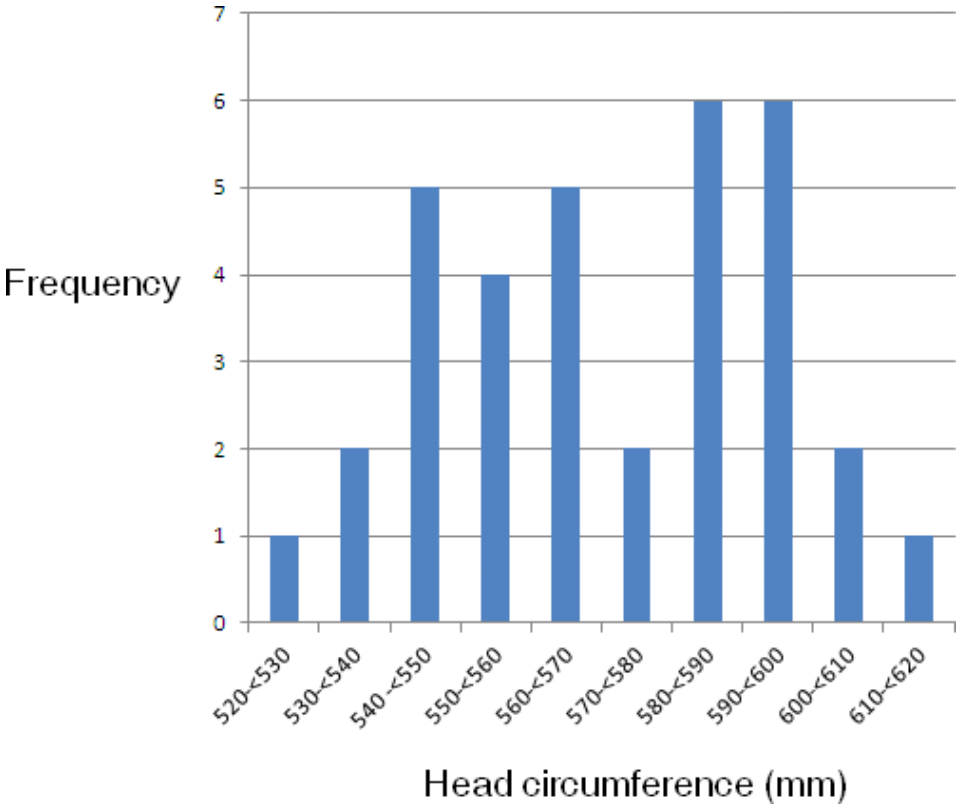
So far you have looked at **line plots** and **stem and leaf plots.** Both are very useful representations for allowing you to see patterns in the variation of your data. A histogram is another useful representation and it is especially useful when dealing with lots of data.

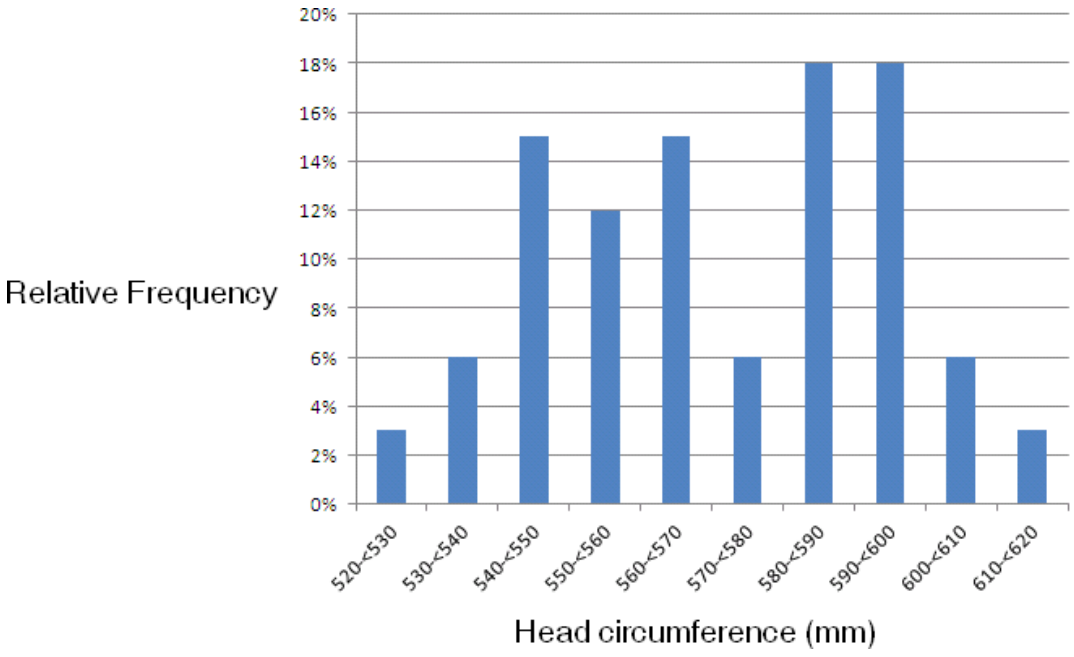
Consider the following:

The frequency and relative frequency for each stem was calculated.

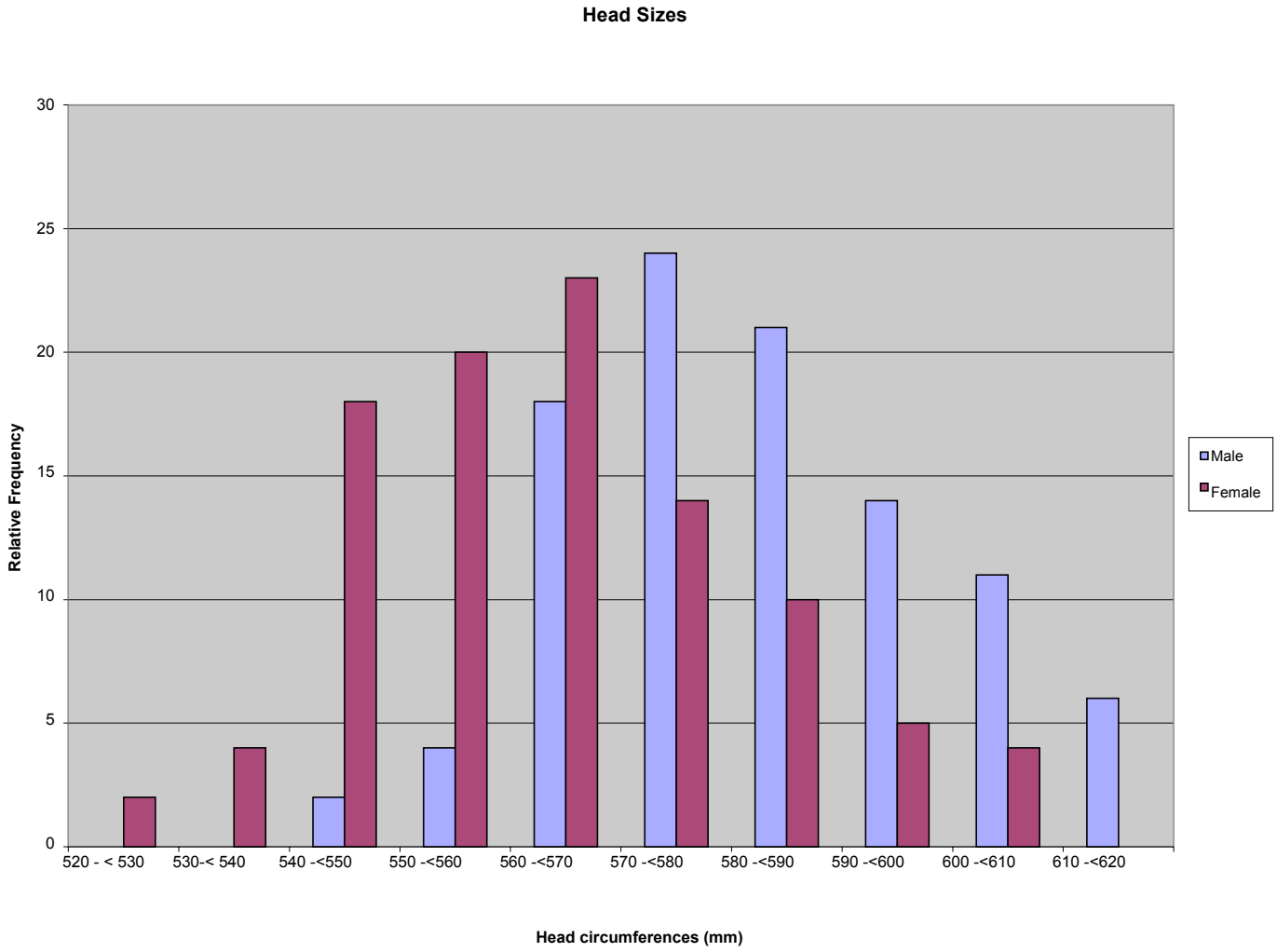
			Frequency	Relative Frequency
52	2	520 - < 530	1	1/34 = 3%
53	1 6	530 - < 540	2	2/34 = 6%
54	0 5 5 5 6	540 - <550	5	5/34 = 15%
55	0 1 2 5	550 - <560	4	4/34 = 12%
56	0 3 4 5 8	560 - <570	5	5/34 = 15%
57	3 7	570 - <580	2	2/34 = 6%
58	0 0 4 5 5 6	580 - <590	6	6/34 = 18%
59	0 0 0 3 4 5	590 - <600	6	6/34 = 18%
60	0 0	600 - <610	2	2/34 = 6%
61	3	610 - <620	1	1/34 = 3%

Using Excel we can draw a histogram. The diagrams below show two representations. Examine the axes. When would it be more suitable to use relative frequency as opposed to frequency ?





Look at the following histogram showing the distribution of head sizes for a different group of males and females. Compare the distributions. Is there any evidence to suggest that there are differences in the head sizes of men and women?



Why do you think the relative frequency is used for this histogram? Does it matter that the actual numbers of males and females in this sample are not given?

Task

Consider the following piece of research.

In the late 1990s a **study** was undertaken in the South Island of New Zealand to explore iron levels in babies and toddlers (age 6-24 months). The participants were selected ***randomly*** from Christchurch, Dunedin and Invercargill (South Island Urban).

The iron, fibre, calcium and vitamin C intake per day was collected over three non-consecutive days and the ***iron (ferritin)*** levels in the blood were measured. Information such as whether the child was being breastfed, fed with formula milk or cows milk, as well as things like gender, ethnicity, maternal education, income level of household, if there were smoker(s) in the household and marital status of the mother were also obtained.

What **type** of study was this? Explain your choice.

Suggest how the participants may have been selected.

Having explored the literature, a number of factors were suggested that could have had an effect on the levels of iron or ferritin in the blood. One of these was gender – boys are at higher risk of having reduced levels of iron or ferritin in their blood.

Some sample data from the study is shown in the tables opposite.

Ferritin Levels	Sex
22.80	1.00
8.00	1.00
	1.00
	1.00
16.80	1.00
	1.00
22.40	1.00
	1.00
17.30	1.00
	1.00
48.20	1.00
27.80	1.00
12.40	1.00
	1.00
5.30	1.00
	1.00
22.20	1.00
39.10	1.00
16.20	1.00
	1.00
18.70	1.00
38.00	1.00
	1.00
8.60	1.00
13.50	1.00
13.00	1.00
5.20	1.00
19.40	1.00
6.21	1.00
	1.00

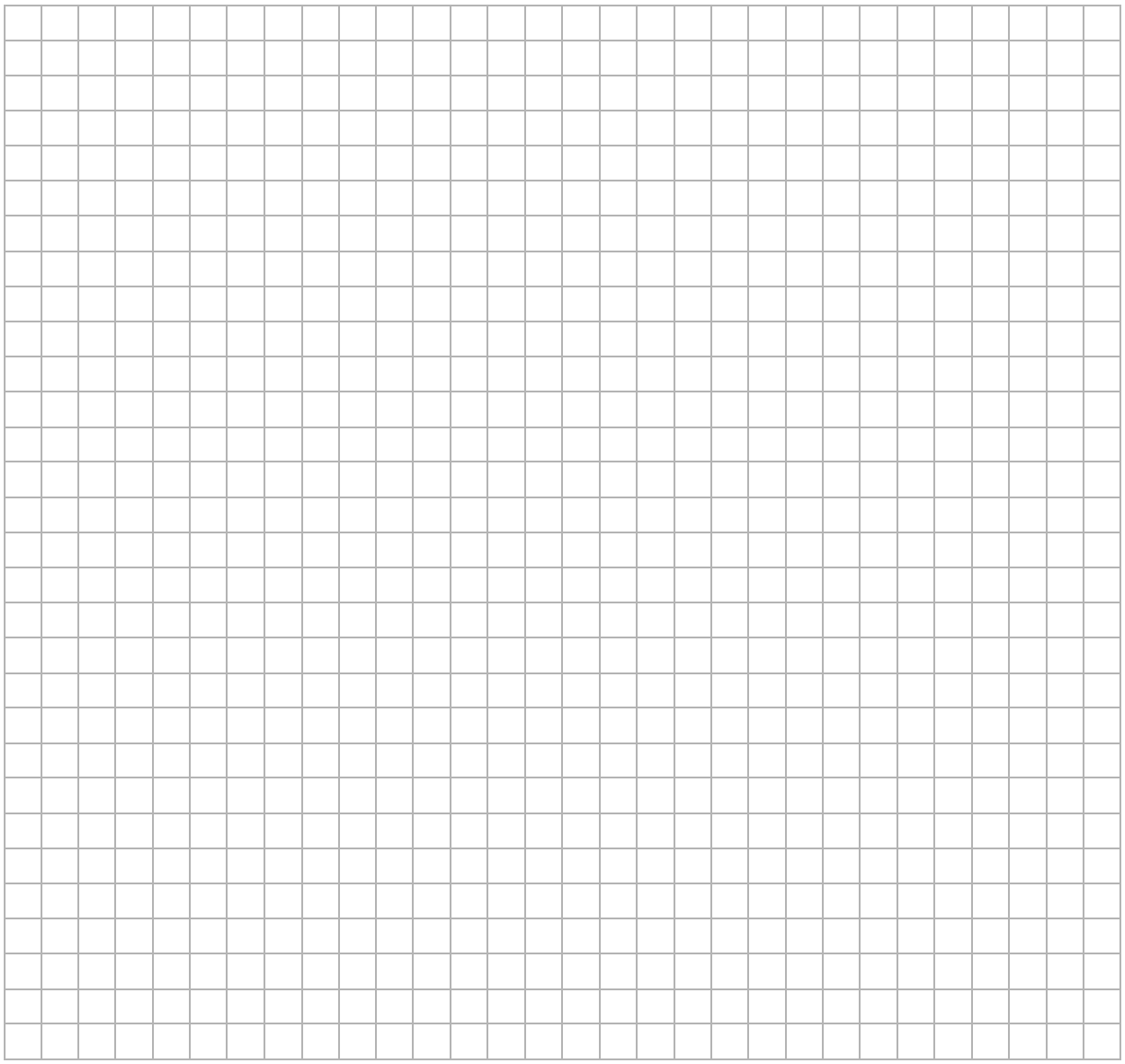
Key

0 = Boy

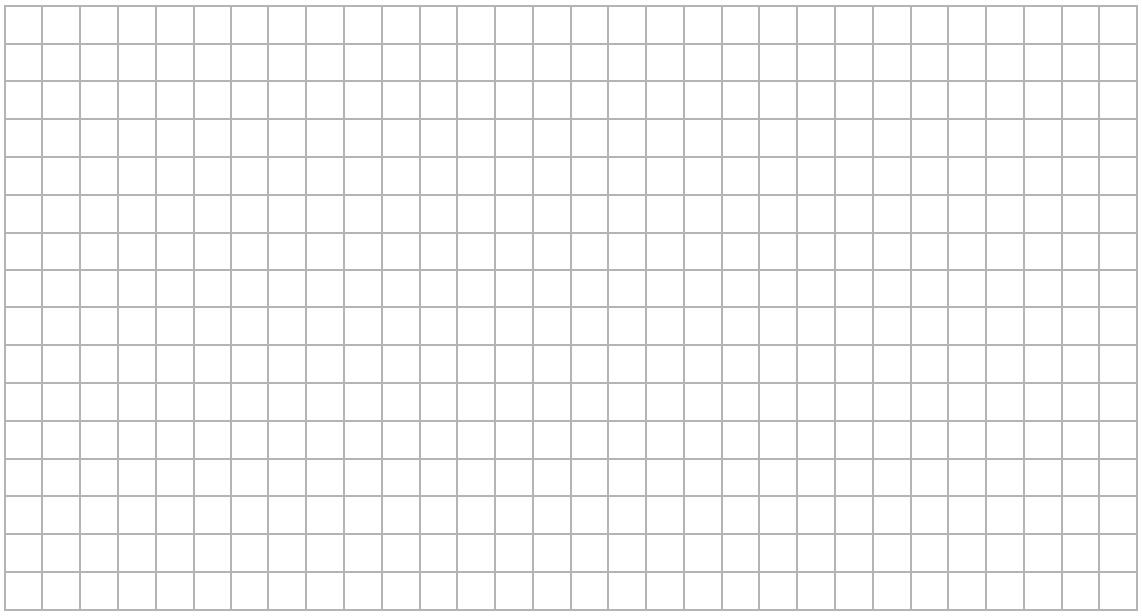
1= Girl

Ferritin levels	Sex
27.20	0.00
	0.00
44.30	0.00
25.00	0.00
17.20	0.00
12.60	0.00
	0.00
9.90	0.00
23.70	0.00
12.75	0.00
	0.00
8.50	0.00
2.00	0.00
32.40	0.00
53.10	0.00
27.50	0.00
16.40	0.00
8.20	0.00
4.80	0.00
16.70	0.00
14.40	0.00
19.50	0.00
20.20	0.00
11.50	0.00
32.70	0.00
	0.00
12.70	0.00
6.30	0.00
10.40	0.00
8.00	0.00

Display this data in a way that will allow you to see patterns in the variation and compare the two **sets of data**



Describe and compare the **shape** of both **distributions of both sets of data**



Describe and compare the **spread** of both **sample distributions.**

Use the data to answer the question “Do the iron levels of South Island urban boys tend to be lower than the iron levels of South Island urban girls?”

Explain why you have made this conclusion.

What aspects of this question did you find confusing?

Was the data difficult to display? What caused this difficulty?

What type of display did you decide to use? How did you make your choice?

How did you deal with the missing data when you displayed your data?

Did the missing data have any impact on the conclusions you made about the study?

Statistics and probability Review

Working through these questions will help you assess your understanding of the learning outcomes listed here:

	All
Learning outcomes	SP.1 investigate the outcomes of experiments so that they can: a. generate a sample space for an experiment in a systematic way, including tree diagrams for successive events and two-way tables for independent events b. use the fundamental principle of counting to solve authentic problems SP.2 investigate random events so that they can: a. demonstrate understanding that probability is a measure on a scale of 0-1 of how likely an event (including an everyday event) is to occur b. use the principle that, in the case of equally likely outcomes, the probability of an event is given by the number of outcomes of interest divided by the total number of outcomes c. use relative frequency as an estimate of the probability of an event, given experimental data, and recognise that increasing the number of times an experiment is repeated generally leads to progressively better estimates of its theoretical probability –

Q. A group of people was asked "***What is your blood type***?" Here is the data they gave.

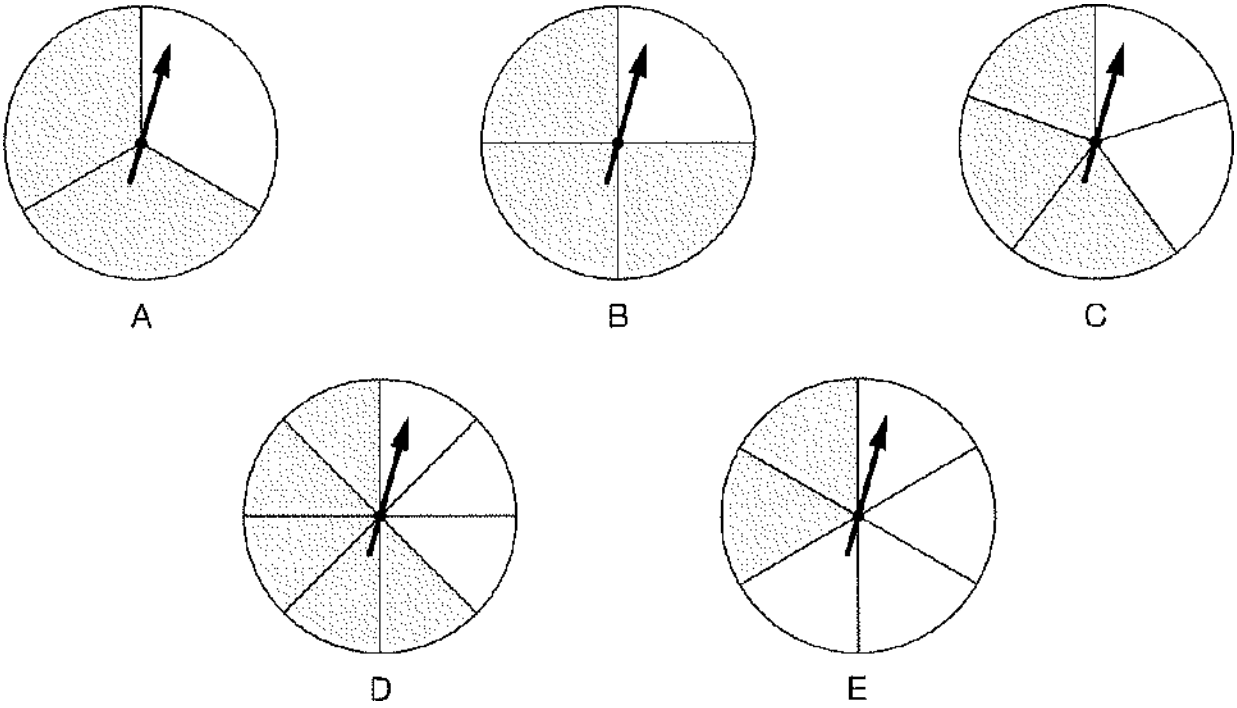
Type A	**Type B**	**Type O**	**Type AB**
50	65	70	15

If a person from this group is selected at random, what is the probability that this person has type **O** blood?

How many people answered the question?
How many people have type O blood?
Remember probability is always a number between **0** and **1.** This means it is a **fraction.** You should write fractions in their **lowest terms.**

Q. Five fair spinners are shown below.

Each spinner is divided into equal sectors, which are coloured either grey or white.



a) Identify the spinner for which the probability of spinning grey is ¾.

b) For two of the spinners, the probability of spinning grey is **more than 60%** but **less than 70%.** Which two spinners are these?

a) If the probability is ¾ what does this mean?

What does the 3 represent? What does the 4 represent?

Can you write ¾ in a different way?

Is 6/8 the same as ¾? Why? Why not?

If a student said the probability of spinning grey was 6/8 what might the spinner look like?

Would the student be correct in saying the probability of spinning grey was 6/8? Why? Why not?

b) Represent 60% and 70% as fractions.

Now work out the probabilities of spinning grey on each spinner.

Can you answer the question now?

Q Two coins are tossed. Complete the diagram to show all the **possible outcomes**.

a) What is the probability of getting 2 heads?

	H	T
T		
H		

b) Jennifer tossed the two coins 50 times and got a head and a tail 28 times.
Is there reason for Jennifer to think that one of the coins is not fair?
Explain.

c) Describe an experiment that would allow Jennifer to determine whether or not the coin was fair.

a) Can you make sense of the diagram? Does it help you to keep track of all the **possible outcomes**? How many possible outcomes are there?

b) Is it **more likely** that you get two heads than two tails? Why? Why not? Is it **more likely** that you get a head and a tail? Why? Why not? If you tossed the coins four times how many times would you expect to get a head and a tail? Why?

c) What would Jennifer have to do? How many times should she throw the dice – twice? 3 times? 100 times?

Remember the learning outcome

Students should be able to

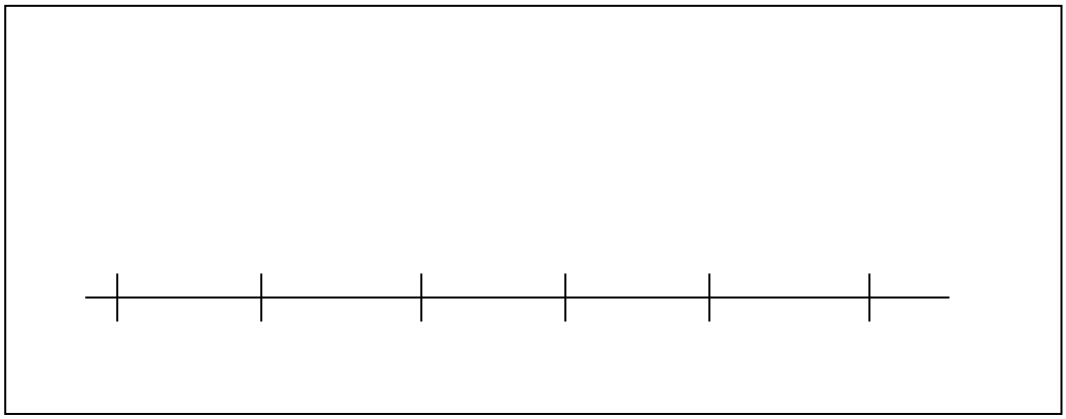
- use relative frequency as an estimate of the probability of an event, given experimental data, and recognise that increasing the number of times an experiment is repeated generally leads to progressively better estimates of its theoretical probability

Q. The table below is a record of the number of texts sent by a group of students in one month.

No of texts sent	0 - 50	50 - 100	100 - 150	150 - 200	200 - 250
Number of students	10	15	25	18	8

a) How many students are in the group?

Illustrate the data on a histogram.



b) Using the table and /or histogram to help you estimate, complete this sentence:

On average these students send about_____texts each month.

c) Sarah is in the group and she sends 210 texts every month. Describe in one sentence Sarah's text sending by comparison to the others in the group.

What is unusual about the way the data is displayed in the table?

If John sends 100 texts in a month where in the table would you enter his data?

What does a **histogram** look like?

What do you think is the **typical** amount of texts sent by students in this group? Does your histogram help you decide on what is the typical amount of texts sent by students in this group?

Q A teacher asked 21 students to estimate the height of a building in metres.

The stem-and-leaf diagram shows all 21 results

6	5	represents 6.5 m

6	5	9				
7	0	2	6	8	8	
8	3	3	5	7	7	9
9	0	5	5	5		
10	4	8				
11	2	7				

a) What is the range of the estimated values?

b) What was the median estimated height?

c) The height of the building was 9.2m. How many people overestimated the height?

What other information can you get form the **stem and leaf plot**?

Is there any **evidence** to suggest that the group are good at estimating building height?

Q. Carol opened a new sandwich bar. She offers a lunch special consisting of a sandwich and a drink for €5.

The different choices available are shown below

Type of bread	Filling	Drinks
Brown	Salad	Tea
White	Egg	Coffee
Wrap	Meat	Hot Chocolate
Panini		Cold drink

All of the different combinations are possible. For example, you can order a salad sandwich on brown bread and a coffee.

How many different lunch specials are possible?

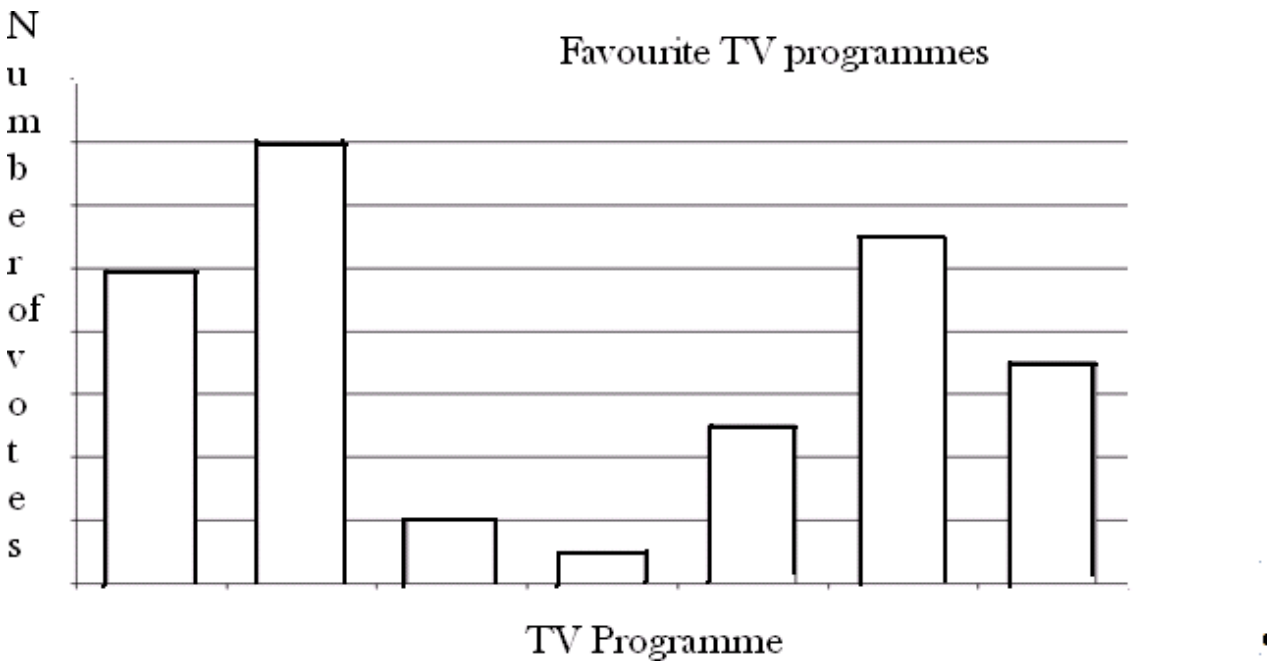
Think of a way to organise your thoughts. Can you write out all the possible combinations? Can you see a pattern as you write out all the combinations? Can you **generalise** this pattern that will help you to find out how many combinations there are without writing them all out?

Q. The lists of test results for two maths classes were posted on the college notice board. You do not know which of the lists is for your class.

List 1	List 2
75	92
80	85
83	87
46	91
35	85
27	81
95	89
84	88
65	87
76	88
15	90
100	92
23	87
20	6
15	0

- Display the data from each list in stem and leaf plots.
- Give one reason why you would hope that list 1 is for your class and one reason why you would hope that list 2 is for your class
- Which list represents the better results? Give a reason for your answer.

Think about what mark you would be hoping to get in the test. Is it **likely** that you would get this mark if your class results were on list 1? List 2?
What is the **typical** mark on list 1? On list 2?
Would you like to get 100? How likely is it that you would get 100 if your class results were on list 1? On list 2? What does it mean to have the **better results**? Is there any evidence that list 1 has **better** results than list 2? Is there any evidence that list 2 has **better** results than list 1?



Clues

- Coronation Street was the most popular TV show
- Twice as many liked Coronation Street as Eastenders
- Fair City got 4 votes less than Coronation Street
- Casualty was the second most popular TV show
- Primetime got 4 votes more than Frontline
- 5 voted for Primetime
- Some people voted for Desperate Housewives

Use the information above to complete the frequency table

TV Programme	No of Votes

Place Coronation street first then Casualty

The bar representing Eastenders must be half the size of the bar representing Coronation Street. Why is this?

Can you locate the bar representing Eastenders?

How will you decide which are the bars representing Primetime and Frontline?

What about the bars representing Desperate Housewives and Fair City?

Task

In 1999 a university librarian put a number of measures in place to try to stop students “stealing” books from the library.

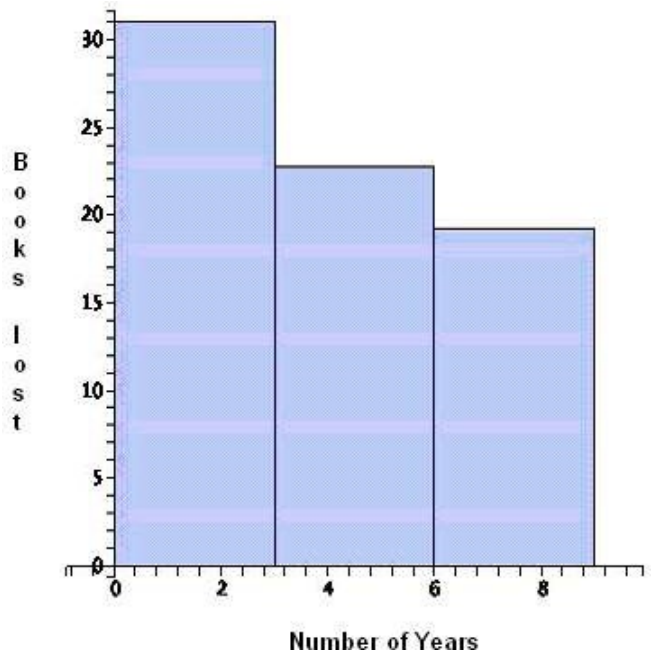
To see how effective these measures were she recorded the number of non-returned books over the next number of years.

The data is recorded below

Year	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**
No of non-returned books	9	10	9	14	6	3	4	5	10

When asked to report to the budget committee on book loss she wrote:

Whilst the drain on resources due to lost books is significant, the histogram below shows that over the last nine years the number of books lost to the library is steadily decreasing, which suggests that the measures implemented to combat this practice are working.



The finance officer was not convinced that the measures were working.

Plot the same data in a histogram but, instead of using three year intervals like the librarian did, divide the data into nine intervals, one for each of the last nine years.

Now, use your histogram to write two statements about the trend.
Does your histogram support the librarian's view that the measures are working, or does it lend more support to the doubts of the finance officer?
Explain your reasoning.

Note to student

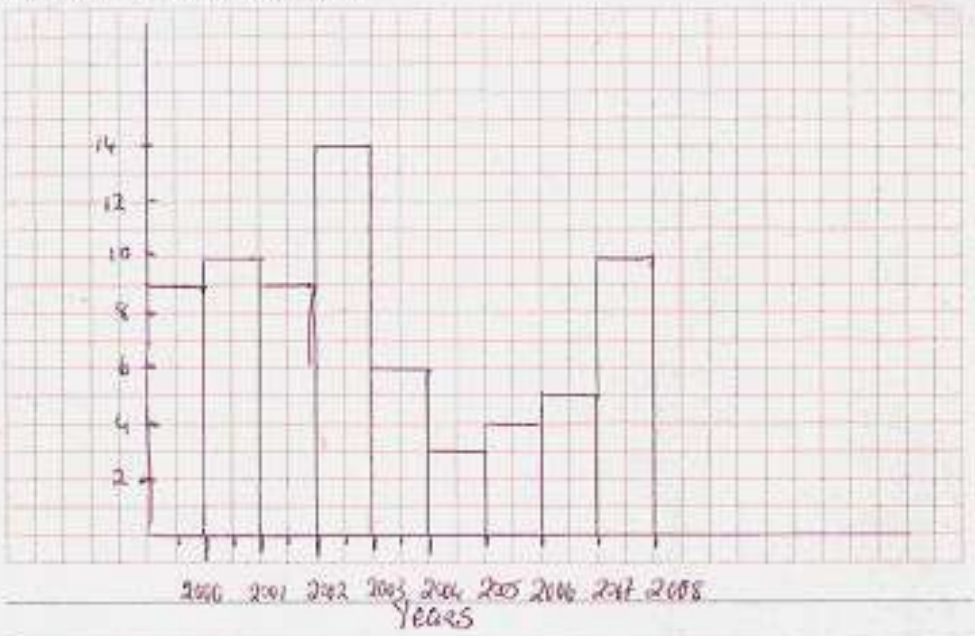
This question highlights the fact that the choice of interval length can **reveal** certain trends or **hide** others.

Look at the plot that is given. How many intervals are there? How many years are in each interval? What can you **conclude** about the number of lost books? Would you say that the measures taken to discourage non-return are working?

Now divide the data into nine intervals and plot the histogram. Is there a difference in the **trend**?

Examine the student work below. Compare this with your work

You may use this page for extra work



This shows a steady increase from 2004 to 2008 which is worrying. There was a good decrease from 2002 to 2004 although it had gone very high from 2001 to 2002

I think this histogram supports the finance officer's view that the measures haven't worked because even though there was a decrease from 2002 to 2004 there is now a steady increase over 4 years and the number of lost books is back to the same as it was in 2001 which was slightly up from when the measures came in

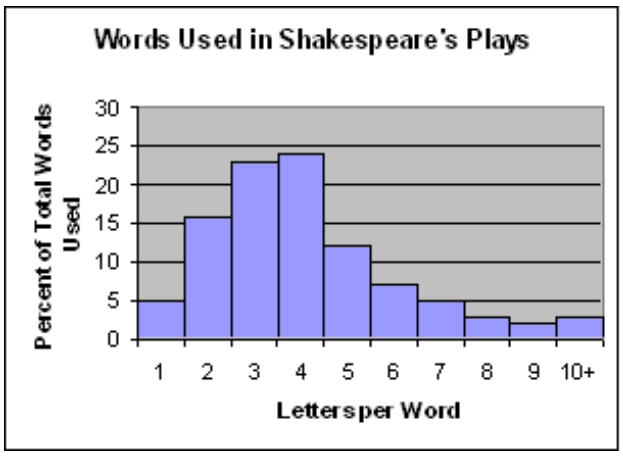
Remember:

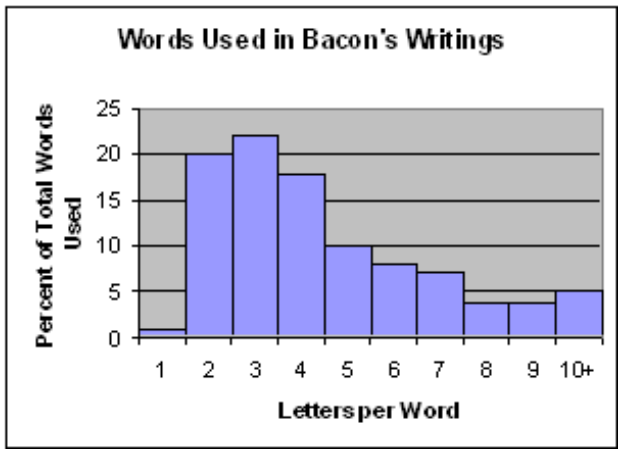
Histograms can be cleverly designed to hide or highlight certain trends. Remember this when you are interpreting histograms.

Which type would give you more detail? Which type would give you less detail?

Why might you want to highlight or hide certain trends?

Q. Some scholars think William Shakespeare was really just a **pen name** for Sir Francis Bacon. (A pen name is a 'fake' name used by another person when writing.) In order to determine if this was true, a researcher counted the letters in every word of Shakespeare's plays and Bacon's writing. The results are recorded in the histograms below.





Based on these histograms, do you think that there is any **evidence** to suggest that William Shakespeare was really just a pen name for Sir Francis Bacon? Explain.

There is a lot of information in these histograms that you could use to support either argument. Yes, William Shakespeare was a pen name for Sir Francis Bacon; or no, William Shakespeare was not a pen name for Sir Francis Bacon. Might there be another explanation?

Are the **distributions** similar? Describe each **distribution.** Use fractions and percentages.

What percentage of Shakespeare's words have 4 letters per word or less?

What % of Bacon's words have 4 letters per word or less?

What percentage of Shakespeare's words have 5 letters per word or less?

What % of Bacon's words have 5 letters per word or less?

Q. The data shows the head circumferences for a group of men and women.

(a) Display the data in a way that will allow you to compare the distributions of head circumferences for both men and women.

(b) Is there any evidence to suggest that men have larger heads than women? Explain your reasoning.

Gender	Head Circumference
F	522
M	580
M	552
F	531
M	563
F	546
F	545
M	545
M	545
M	568
F	560
M	613
F	555
F	573
M	577
F	584
M	600
M	595
M	593
F	590
M	594
F	564
F	536
M	586

There are a lot of ways to display this data; a line plot, a back to back stem and leaf plot, or a histogram. Eyeball the data and think about how you would display it.

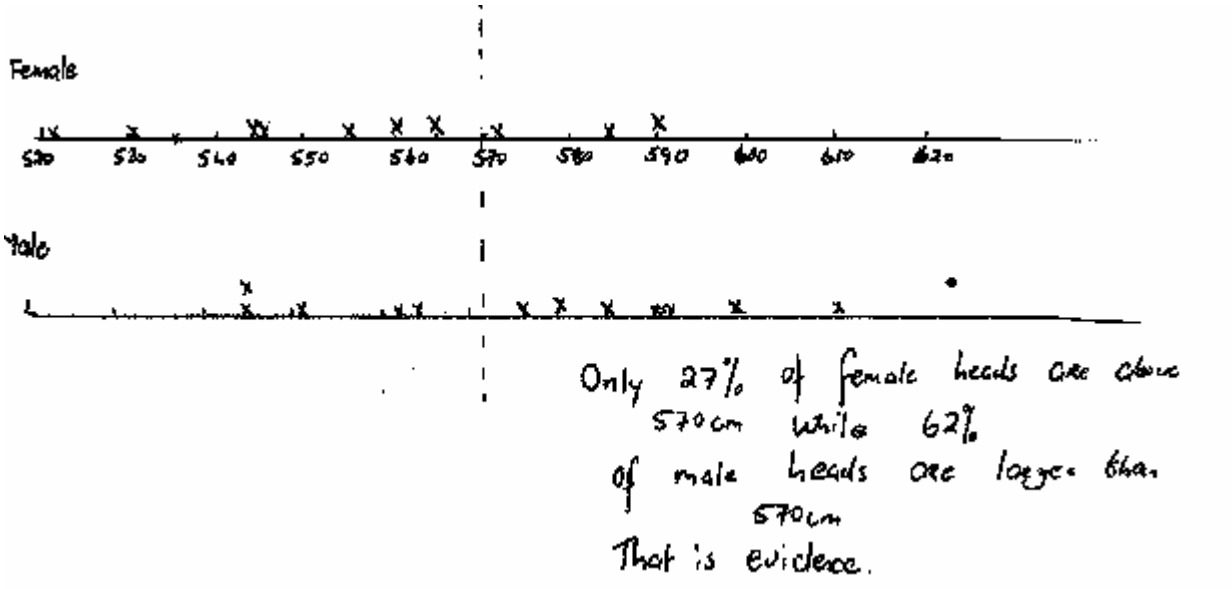
- What features are you looking at in the data?
- How are you deciding which display is most appropriate?

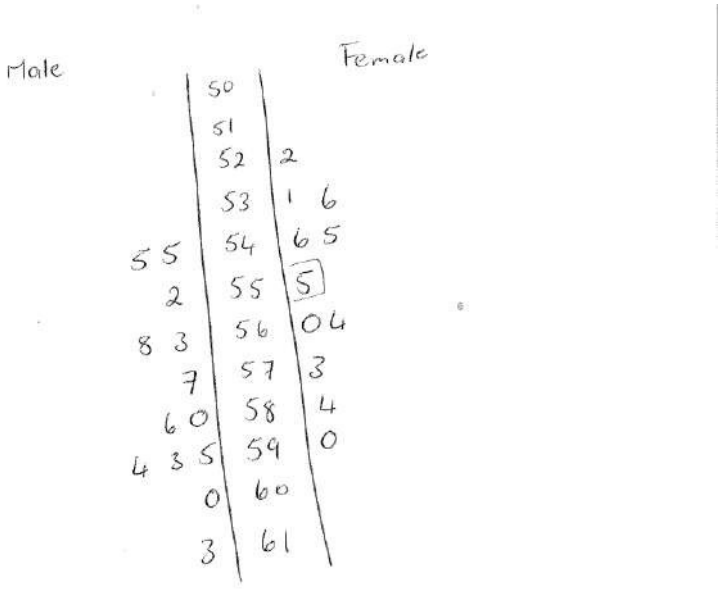
Once displayed you will be able to **comment** on the **distributions** and **draw conclusions** about the relative sizes of the heads of men and women.

Have a go at this and then examine the following examples of how other students displayed the data and drew conclusions from it.

17

Student work





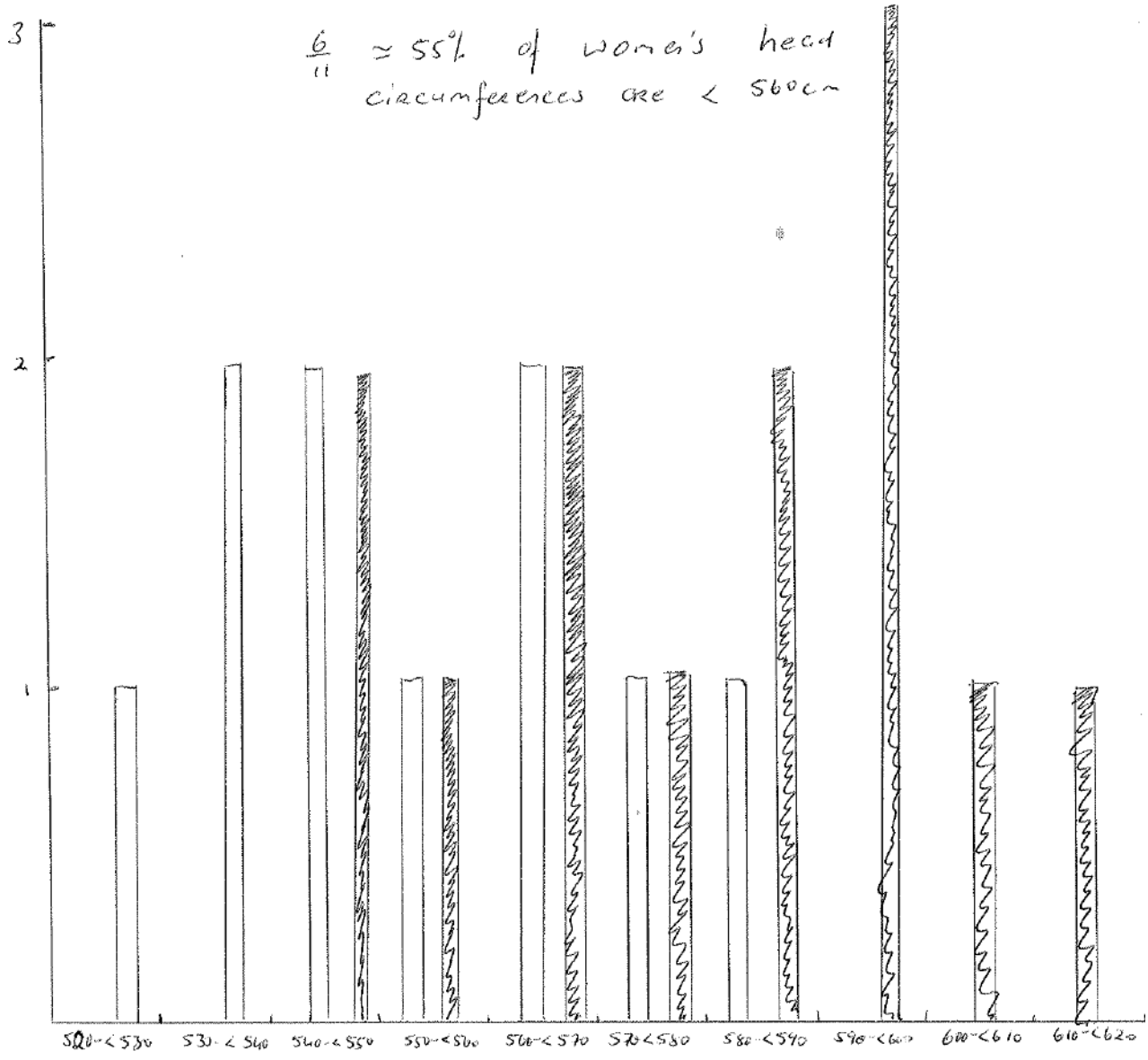
Range of female is 590 - 522 = 68
Range of male is 613 - 545 = 68

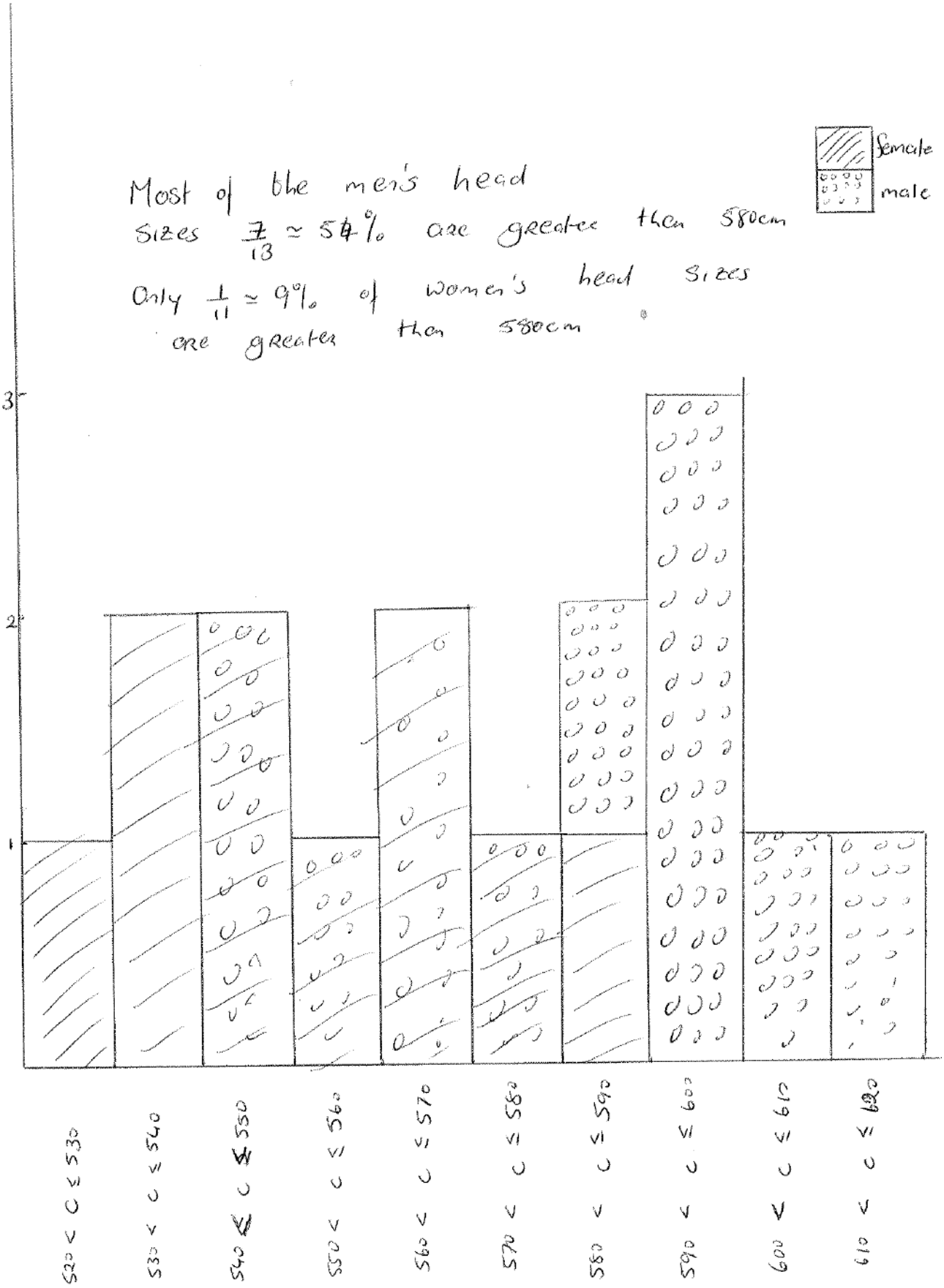
50% of the female head sizes lie between 522cm and 555cm while only 23% of the males lie in that Range
54% of male head sizes are between 580 and 613cm and only 18% of female head sizes are in this Range so yes there is evidence to support statement men have biggest heads than women

	Male	Female
$520 < C \leq 530$	0	1
$530 < C \leq 540$	0	2
$540 < C \leq 550$	2	2
$550 < C \leq 560$	1	1
$560 < C \leq 570$	2	2
$570 < C \leq 580$	1	1
$580 < C \leq 590$	2	1
$590 < C \leq 600$	3	0
$600 < C \leq 610$	1	0
$610 < C \leq 620$	1	0

$\frac{7}{13} \approx 54\%$ of men's head circumference is $> 580cm$

$\frac{6}{11} \approx 55\%$ of women's head circumferences are $< 560cm$




female
male
Most of the men's head sizes $\frac{7}{13} \simeq 54\%$ are greater than 580cm
Only $\frac{1}{11} \simeq 9\%$ of women's head sizes are greater than 580cm
3
2
1
520 < c ≤ 530
530 < c ≤ 540
540 < c ≤ 550
550 < c ≤ 560
560 < c ≤ 570
570 < c ≤ 580
580 < c ≤ 590
590 < c ≤ 600
600 < c ≤ 610
610 < c ≤ 620

Q. The 5th year and 6th year students in a local school were asked about the number of hours per week they spent playing on a games console. The results are shown below.

Number of hours spent playing on a games console	**Number of 5th year students**	**Number of 6th year students**
1		
2	1	1
3	2	3
4	1	1
5	1	2
6	5	2
7		3
8		
9	1	3
10		1
11		3
12		2
13	3	3
14	1	1
15	4	
16	4	3
17	2	1
18	4	2
19	4	4
20	3	2
21	2	
22	3	
23	1	
24		
25	1	4

Display the data in a way that allows you to **comment** on the **shape of the distributions**. Is there any **evidence** to suggest that 6th year students spend longer playing a games console than 5th year students?

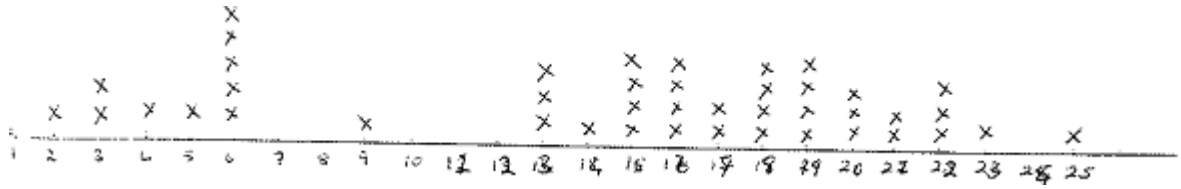
Note to Students

There are many ways you may choose to answer this question.
The data could be displayed in line plots, a back to back stem and leaf plot, or a histogram.
Once displayed you will be able to **comment** on the **distributions** and **draw conclusions** about the relative times spent by 5th and 6th year students on games consoles.

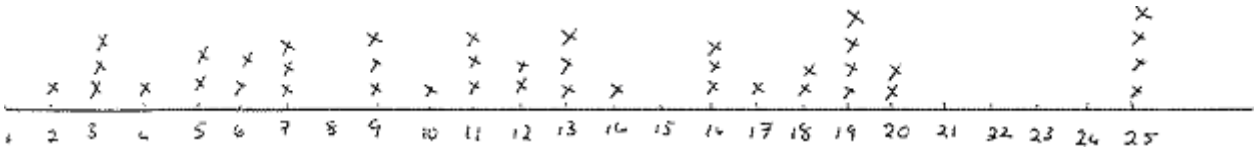
Have a go at this and then examine how student A below displayed the data and drew conclusions from it.
Now try to use a back to back stem and leaf plot and a histogram. Evaluate each display.

Student A

5TH Year



6th year

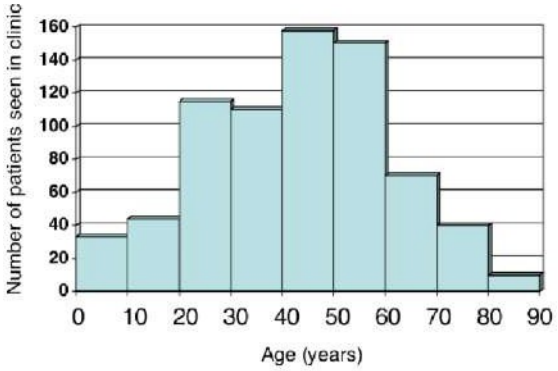


In the 5th year data there are two clusters: between 2 and 6 hours per week and 13-23 hours per week. 10 out of 43 or almost 25% of students play the console over the range of the first cluster. 31 of 43 or 72% are in the second cluster.
Only 1 out of 43 students uses the games console between 7 and 12 hours per week.

The data from 6th years are more evenly spread than the 5th year data there are no real clusters. 16/41 or 39% of students play the games console between 16 and 25 hours per week while ½ of the students play between 2 and 12 hours per week.
The range is the same as the 5th years.

The fact that 24 of 43 or approximately 56% of 5th year students play the console between 16 and 25 hours per week whilst only 39% of the 6th year students play for this length of time indicates that there Is no evidence to suggest that 6th year students spend longer playing a games console than 5th year students. In fact the evidence shows the opposite.

Q. The ages of the patients seen by a group of doctors in a clinic over the last month are shown in the histogram below.



The clinic is about to begin a Swine Flu vaccination programme and must order the drugs they need from the HSE.

If $^1/_3$ of the 40-90 year olds, ½ of the 20-40 year olds, $^1/_5$ of the 10-20 year olds and all the 0-10 year olds who attended the clinic last month are likely to attend for vaccination, what is the minimum number of vaccinations that the clinic should order from the HSE?
Show your workings.

Use the histogram to decide how many of each age group visited the clinic over the last month.
Try to organise your work into a table
How many 40-90 year olds attended the clinic in the last month? What fraction of these is likely to attend for vaccination? How many of these are likely to attend for vaccination?
What about the 20-40 year olds? What fraction of these attended the clinic in the last month? How many of these are likely to attend for vaccination?

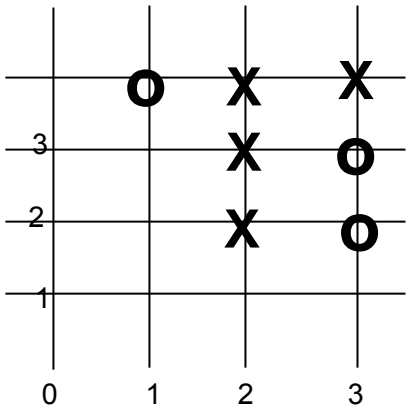
Set A: Review Materials – Junior Cycle

Statistics

This set of questions, compiled in two documents, is intended to help you review your work as you prepare for the Junior Cycle examination. The questions are not intended to be exact matches of what will come up in the exam but they should give you a flavour of how the concepts can be examined in context.

Q Melissa and Sean are playing a game

Melissa has to make a line of 4 **X** towin.



Put an **X** on the grid to make a winning line for Melissa

Write the co-ordinates of each **X** in this winning line.

(….. , …..) (….. , …..) (….. , …..) (….. , …..)

Look at the numbers in the co-ordinates of these points.

What do you notice?

Is the point (1, 6) on Melissa's winning line?

How do you know?

Where can you put the X so that there are 4 in a row?

Try out different places ….you may extend the grid if you like. Now decide where you would put the X so that there are 4 in a row.

Now try to remember how to label points on a co-ordinate grid. How far did you go out along the x axis? This is the x-coordinate.

How far did you go up or down along the y axis? This is the y-coordinate.

Can you see a pattern between the x and y coordinates?

It might help if you were to put them in a table

x-coordinate	y-coordinate

Now think about the point (1, 6) is this on the winning line? How would you know?

One way to find out is to put the point (1, 6) in your table and see does it fit with the pattern you saw before.

If it doesn't fit with the pattern you saw why do you think this is? Try to explain.

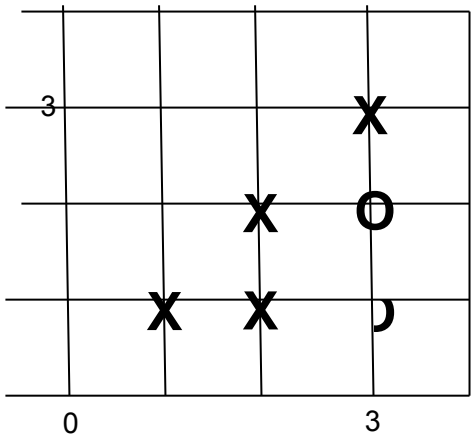
Can you think of another way to make a decision about whether or

Q.Mags and Seamie are playing the game

Mags has to make a line of 4 **X** to win

Put an **X** on the grid to make a winning line for Mags.

Write the co-ordinates of the four **X** in this winning line.



(….. , …..) (….. , …..) (….. , …..) (….. , …..)

Look at the numbers in the co-ordinates of these points. What do you notice?

Is the point (6, 7) on Mag's winning line?

How do you know?

What is the relationship between the x and y coordinates of all points on Mag's winning line?

Where can you put the X so that there are 4 in a row? What is different about this question and the question above?

Try out different places ….you may extend the grid if you like. Now, where would you put the X so that there are 4 in a row?

Now try to remember how to label points on a co-ordinate grid. How far did you go out along the x axis? This is the x-coordinate.

How far did you go up or down along the y axis? This is the y-coordinate.

Can you see a pattern between the x and y coordinates?

It might help if you were to put them in a table

x-coordinate	y-coordinate

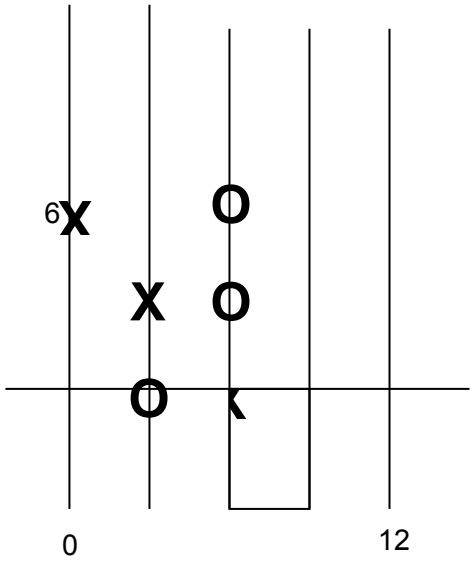
Now think about the point (6,7); is this on the winning line? How would you know?

One way to find out is to put the point (6,7) in your table and see does it fit with the pattern you saw before.

If it doesn't fit with the pattern you saw why do you think this is? Try to explain

Q.Michael and Sam are playing a game

Michael has to make a line of 4 **X** towin.



Put an **X** on the grid to make a winning line for Melissa

Write the co-ordinates of the four **X** in this winning line.

(….. , …..) (….. , …..) (….. , …..) (….. , …..)

What is the relationship between the x and y coordinates of all points on Michael's winning line?

If the x coordinate of a point on this line is 25 what should the Y coordinate be?

Explain how you arrived at your answer.

Scaling the axes is a challenge in this question, look at the axes and see why this is the case. What is different about this question and the two questions above?

Can you see a pattern between the x and y coordinates?

It might help if you were to put them in a table

x-coordinate	y-coordinate

Now think about other points on this winning line; they should fit with this pattern. Try to generalise the pattern you see; this will give you the equation of the line. Can you find the equation of the line in any other way?

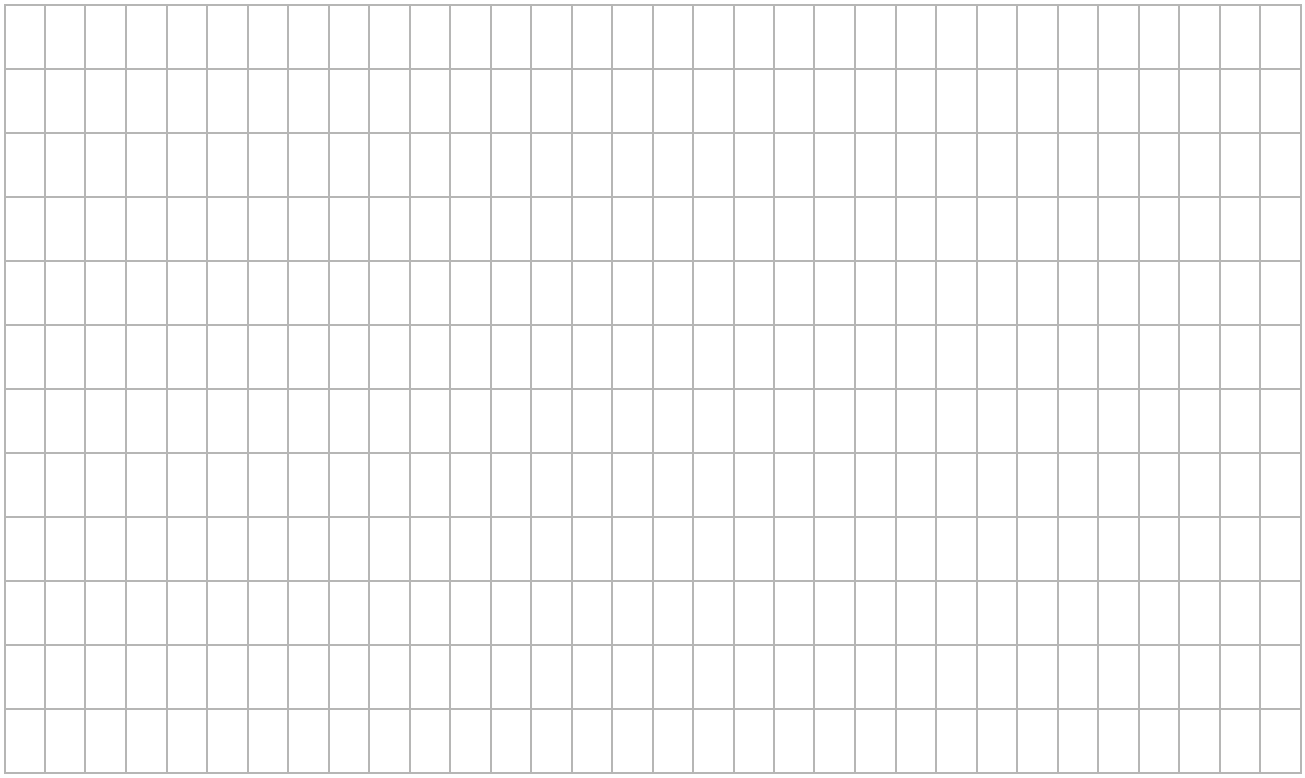
Compare the two methods.

Remember the equation of a line is just the generalisation of the pattern that exists between the x and y coordinates of the points on a line. Once you know this generalised pattern you can find any points on the line and make predictions about the line.

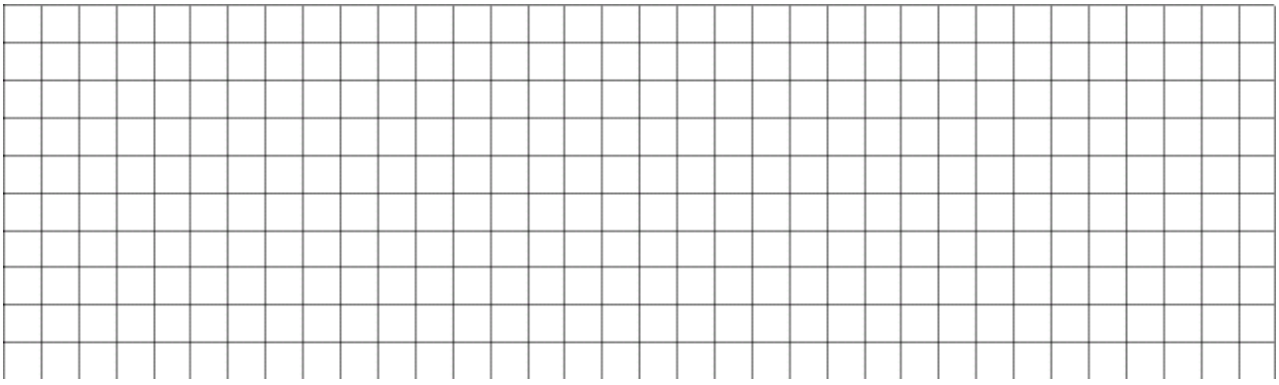
Q. Joe and Sophie were investigating the relationship between the current flowing through a wire and the voltage across the wire. They performed an experiment and recorded their results in the table.

Voltage (Volts)	Current (Amps)
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6

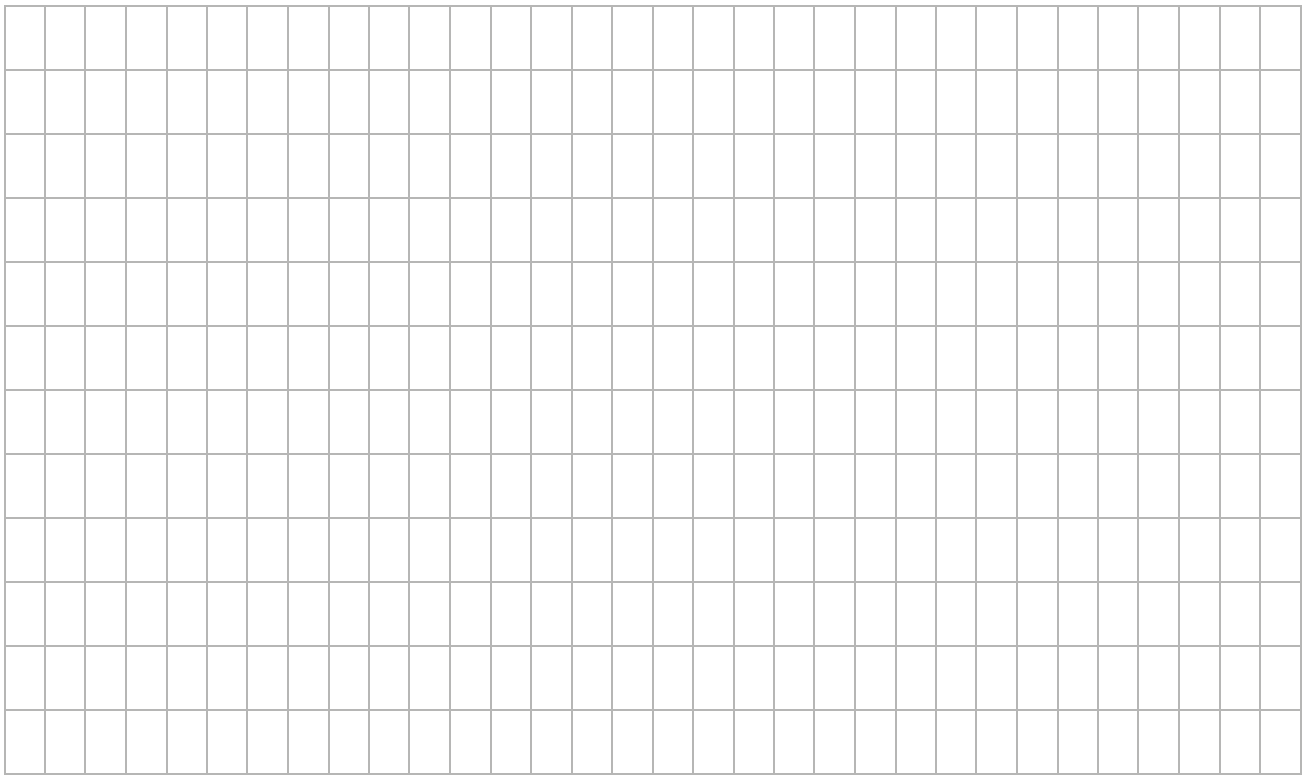
Plot their results on a coordinate grid.



What is the relationship between the x and y coordinates? Generalise this relationship and write it in the form of an algebraic formula.



If the voltage across the wire was 10 volts, what do you think the current flowing through the wire would be? Explain your thinking.



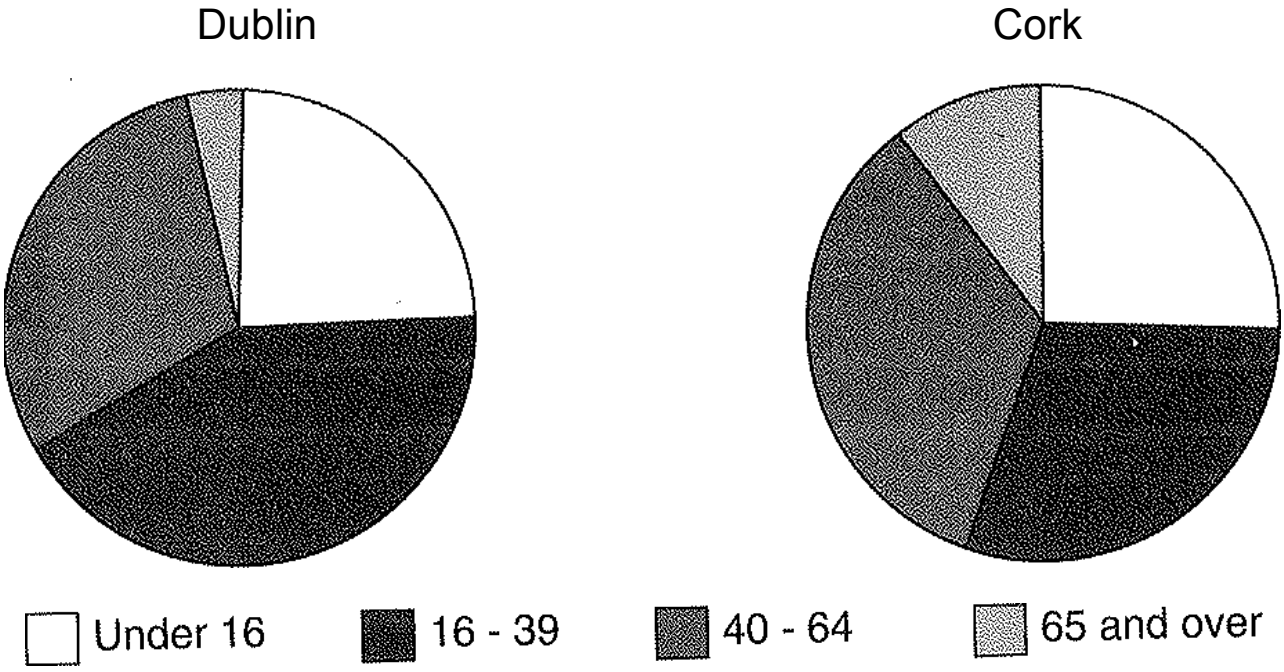
When you plot your points on the grid decide what type of a relationship exists between the current flowing through the wire and the voltage across it. Is it a linear relationship? How would you know? Is it a quadratic relationship? How would you know? Is it an exponential relationship? How would you know?

Click on the *concept of slope* presentation for help with this question.

When you have decided on the type of relationship that exists between the current flowing through a wire and the voltage across it you can generalise this relationship; again the *concept of slope* presentation should help you with this.

Once you have generalised the relationship or know the equation you can answer lots of questions about the relationship between other points that lie on the line.

Q. These pie charts show information from a survey of people's ages. 400 people in Dublin and 800 people in Cork were surveyed.



John looked at the charts and said

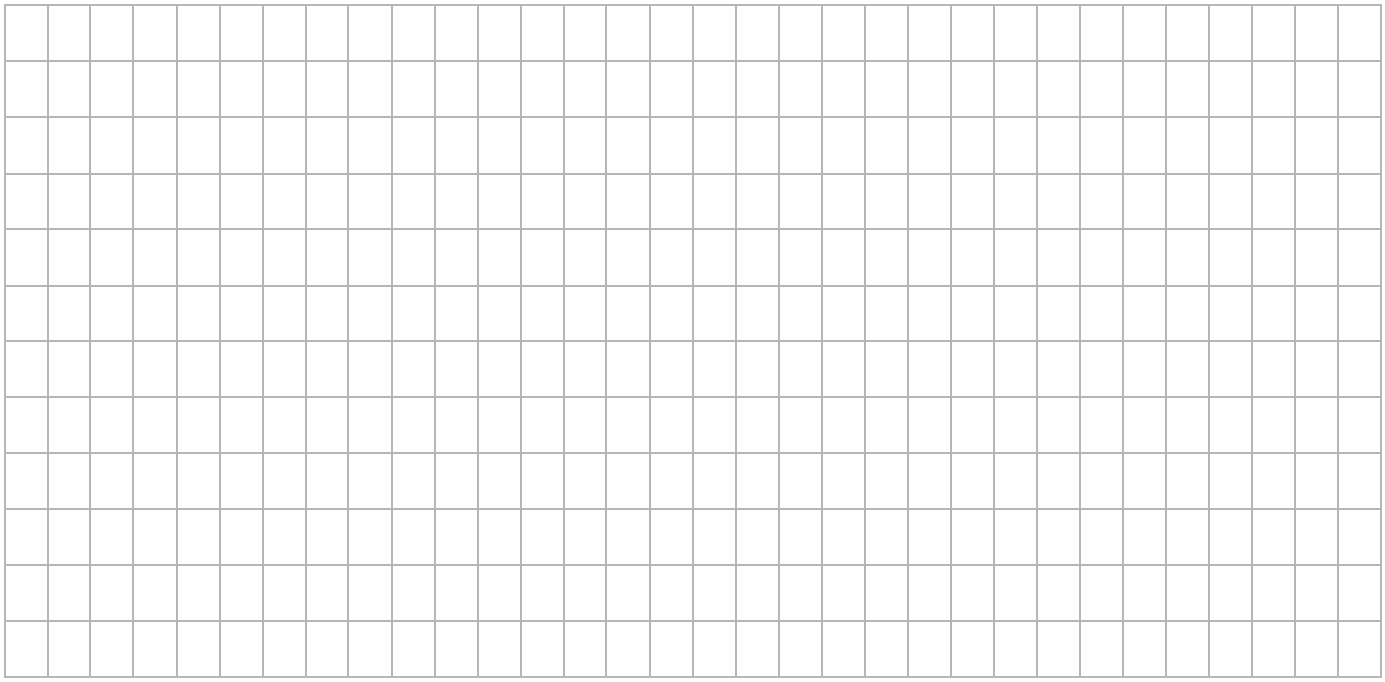
"There is roughly the same number of people under 16 in Cork as there is in Dublin"

Looking at the charts, why do you think John said this?

Do you agree with John? Give a reason why you do or do not agree with him.

Display the data in a way that will make it easier for John to compare the two surveys.

Explain why you made this choice.



This question was designed to promote discussion about pie charts and the information that they can give you. When you discuss things with your friends it gives you an opportunity to get a good idea about what they are thinking in their heads. Sometimes you are all thinking the same thing; sometimes when you hear what others think it makes you think again about your own ideas .You might say “Gosh I never thought about it like that” or “I never really knew that”; when this happens you are able to **refine** your ideas to take into consideration those of your friends. At other times you might disagree and think “No that is not what this is about” and you will **defend** your ideas to your friends. Both of these types of reactions, **reflection/refinement** and **defending,** are a very important part of the learning experience. When your teacher engages in discussion with you he/she gets an idea of what is in your head and he/ she will be able to help you change/refine or extend your thinking. That is why you will find you are doing a lot more discussing these days in Maths class.

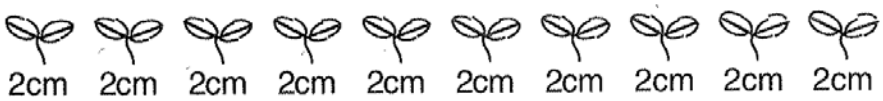
Now back to this question. Do you agree with John? Exactly what information is contained in the sections of a pie chart? Does it contain exact amounts? or proportions? If it contains exact amounts, then is John right? If it contains proportions then is John right? Can you see why John may or may not be right? Is the fact that 400 people were surveyed in Dublin and 800 surveyed in Cork significant? If so, how?

Q. An advert says

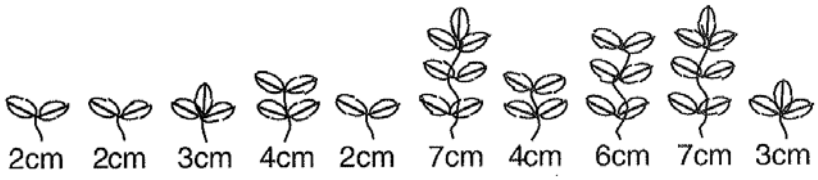
Wondergrow doubles the height of your plants in 2 weeks

Susie uses Wondergrow on her plants.

Day 1 (at the start)



2 weeks later



a) Does Wondergrow really double the height of the plants?

Use the **mean** and **range** to explain your answer.

b) If Susie chose one of her plants at random what is the probability that it would have doubled in height after 2 weeks?

c) Circle the statement which you think is most accurate. Use Susie's data to explain your choice.

i. It is **impossible** that your plants will double in height after 2 weeks if you use Wondergrow.

ii. It is **unlikely** that your plants will double in height after 2 weeks if you use Wondergrow.

iii. It is **likely** that your plants will double in height after 2 weeks if you use Wondergrow.

iv. It is **certain** your plants will double in height after 2 weeks if you use Wondergrow.

This question encourages you to think about statistical claims and to use evidence from data to agree with or disagree with a claim.

Take a first look at the data; what are you first instincts? Does Wondergrow double the height of any of the plants? All of the plants? Some of the plants?

What does the **mean** height tell you? Calculate the **mean** height before and after the treatment with Wondergrow. What has Wondergrow done to the **mean** height of the plants?

What about the **range** of heights? What was the **range** of heights before the treatment with Wondergrow? and after?

What does the **range** tell you about the heights of the plants?

Looking at the data; how likely is it that if you use Wondergrow it will double the height of your plants after 2 weeks?

Certain? Why? Why not?

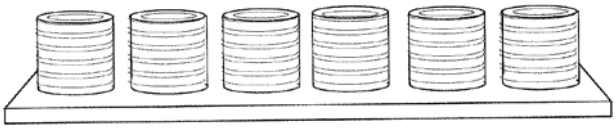
Impossible? Why? Why not?

Likely? Why? Why not?

Unlikely? Why? Why not?

Q. Kai has 6 tins of paint but the labels have come off.

He knows that he had



White, Magnolia, Yellow, Rose, Midnight Sun, and **Cream**

Kai picks a tin.

He thinks that the probability that he will pick **magnolia** is 1/6.

Kai is **right.**

Explain why.

Yetunde knows that students in his school liked to watch these sports:

Soccer Gaelic Football Rugby

He says:

"*The probability that the next person I meet likes to watch Rugby is 1/3, because there are three sports*"

Do you agree with Yetunde? Explain why.

Think! How many cans of Magnolia paint are there?

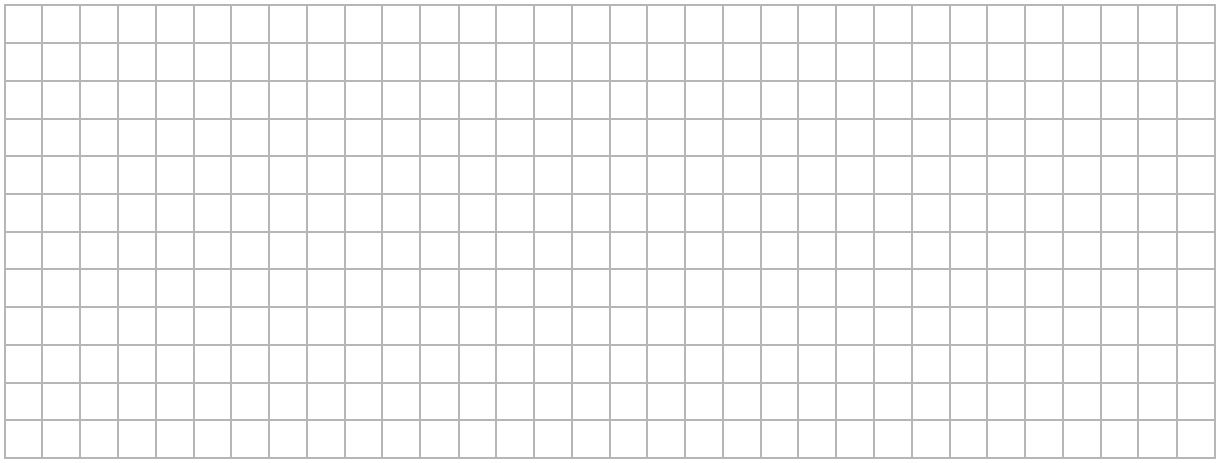
How many cans of paint are there altogether?

Can you see now why Kai is right when he says the probability of choosing a can of magnolia paint is 1/6?

Think about your school; if you wanted to know the probability of a student liking soccer, rugby or Gaelic football how would you go about finding out?

Would you have to survey the students? Or would you agree with Yetunde there is no need to survey the students because there are three sports so the probability of someone liking soccer must be 1/3?

Q. During May 2010, 110 cars were taken to a car testing station. The results showed that 36 had defective brakes and lights, 42 had defective brakes, and 47 had defective lights. A car will not pass the test if it has one or more of these defects.
Display the information in a Venn diagram.



What is the probability that a car chosen at random

a) Failed the test

b) Passed the test

c) Had exactly one defect.

Q. Sarah, Jo, Alan and Amy want to find out what people think and do about child labour.

They are preparing a questionnaire.

Here are some questions they suggest:

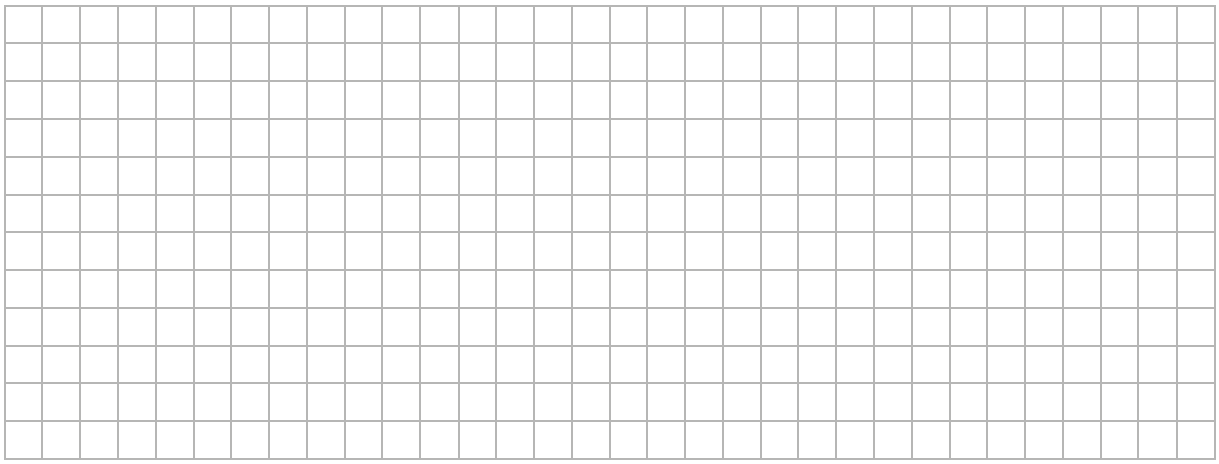
Sarah: Are you a member of a human rights organisation? Yes/No

Jo: Are children important? Yes/No

Alan: Don't you agree that making young people work is very, very cruel? Yes/No

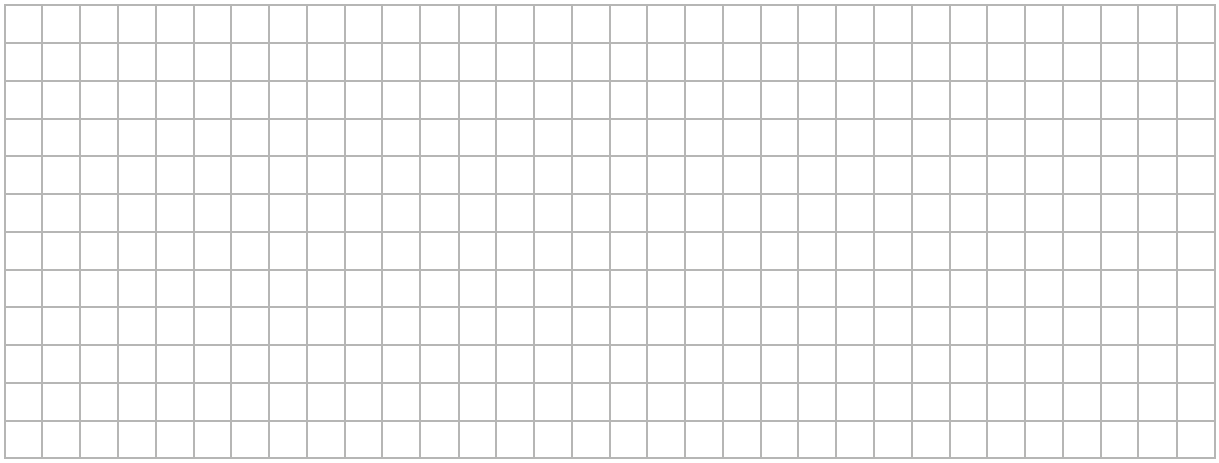
Amy: Do you buy products from shops that sell goods manufactured by children? Yes/No

Choose two of these questions that **you** think ***should not be used***. Whose questions are they?



Explain why you think these two questions should not be used

Write an extra question that **you** would use. People should be able to answer the question with ‘Yes’ or ‘No’.



Q. A librarian asks you to do a survey of the people using the library. She does not want you to talk to any of the members.

She chooses these five headings:

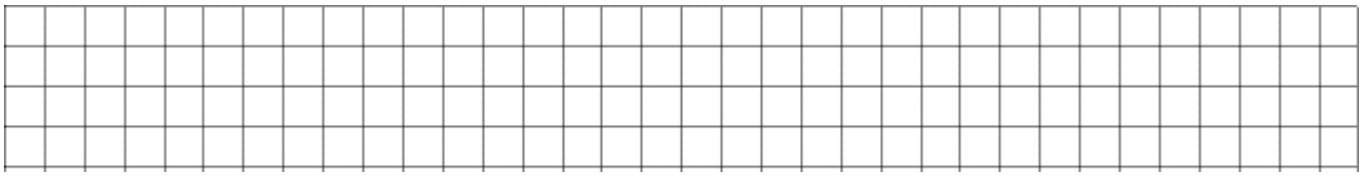
Time of day, Sex, Age, What was borrowed, Reason for borrowing the item

She gives you this record sheet:

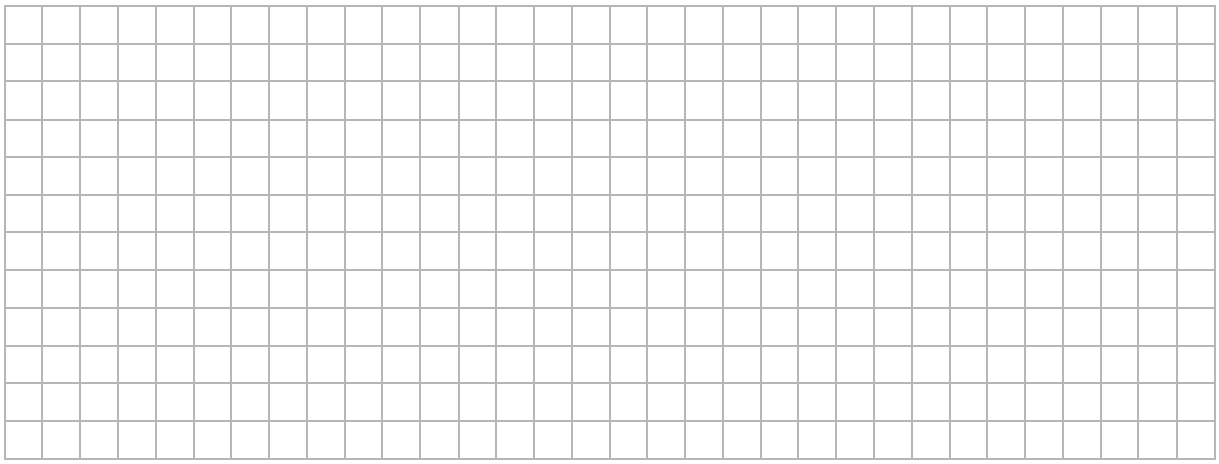
Time of day	Sex	Age		What was borrowed				Reason for borrowing the item
	M/F	Under 40	40 or over	CD	DVD	Novel	Ref Book	

It is easy to use the heading "**Sex**" because it is usually easy to see if a person is male or female.

Pick one heading which is harder to use for collecting information.



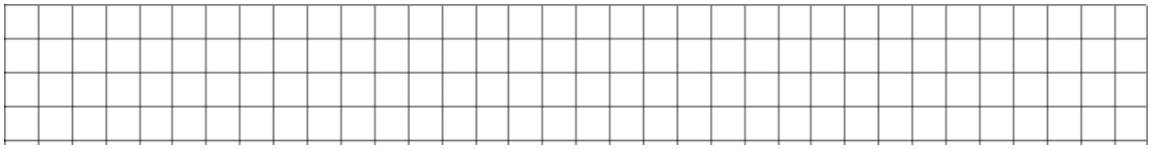
Explain why it is harder to use



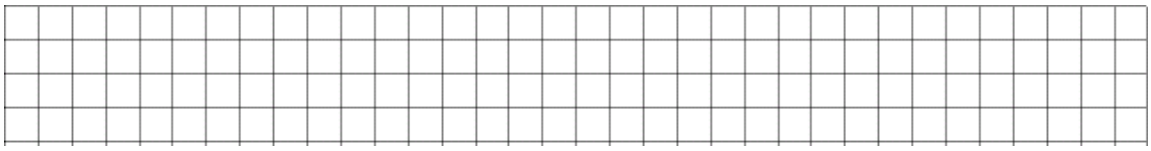
For the rest of this question you will need to think about surveys which **you** have done.

Think back to a sheet which **you** have used to collect information.

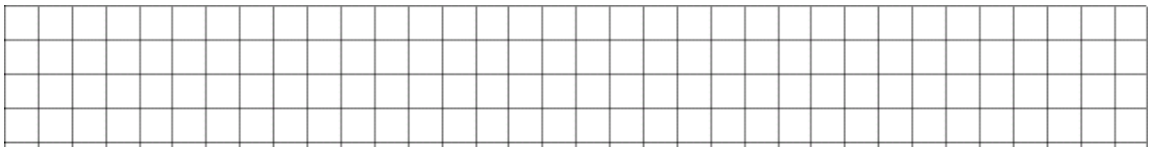
What were you collecting information about?



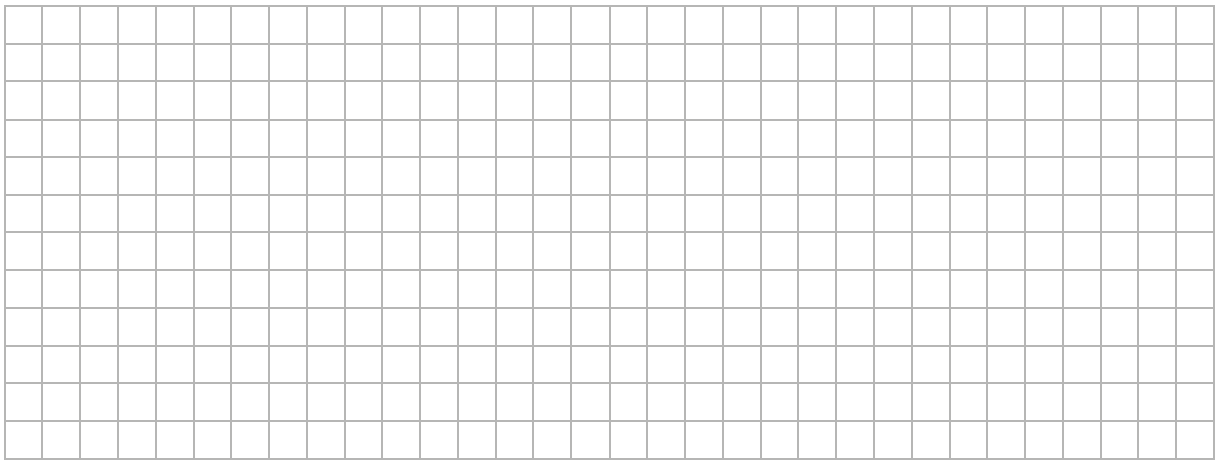
Write down one heading you used. It can not be the same as any of the librarian's headings.



Was the heading easy to use for collecting information?



Explain why.



Think about designing questionnaires. It is likely that you have done a statistical investigation in class and may have had to ask people questions in order to get information or data. **Bias** is something you should always consider when you are asking people questions. The way you ask the question can influence the answers that people give, this is known as bias. If you ask a biased question your data is **unreliable** and you can't really be sure that is what the person who answered really thinks.

Q. The youth club is planning a trip This is what each person chooses

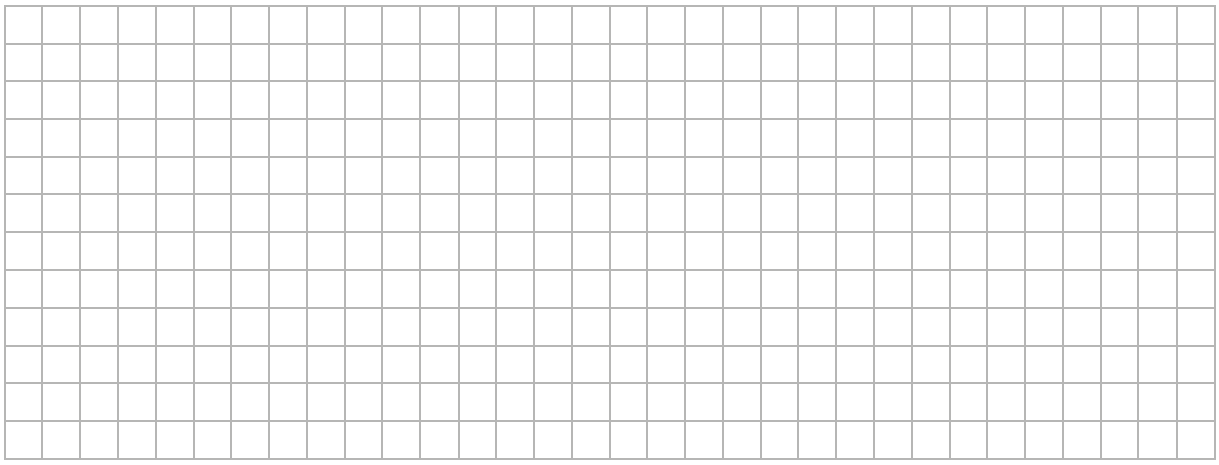
Cinema	Sarah, Amy, Mags, John, Eamonn, Sean, Padraig, Mary, Steven, Anne, Erica, Paul
Bowling	Ross, Charlie, Roy Bernie, Amanda, Adrian, Hannah, Erin
Quasar	Brendan, Pete, Lauren, Gavin, Paul, Ciaran

Display this data in a way that will allow you to answer the questions below.

Where do most people want to go?

The Youth leader decides to ask everyone to write their choice on a piece of card and places these in a hat.

The Youth leader pulls 1 piece of card from the hat. This is where they will all go. What is the probability that Adrian will get his choice?



Q. Rosin and Peter wanted to see which of the two restaurants in town gives the best value for money.

They decided to visit each restaurant over a two-week period, order a meal and record the number of chips on their plates. The results are recorded below

Lucy's Lunches	Number of chips on the plate													
	33	34	34	35	34	32	34	33	36	30	32	33	34	35
Dave's Diner	39	26	25	42	35	47	42	39	24	30	37	42	26	25

Display the results in a way that will allow you to compare the two sets of data.

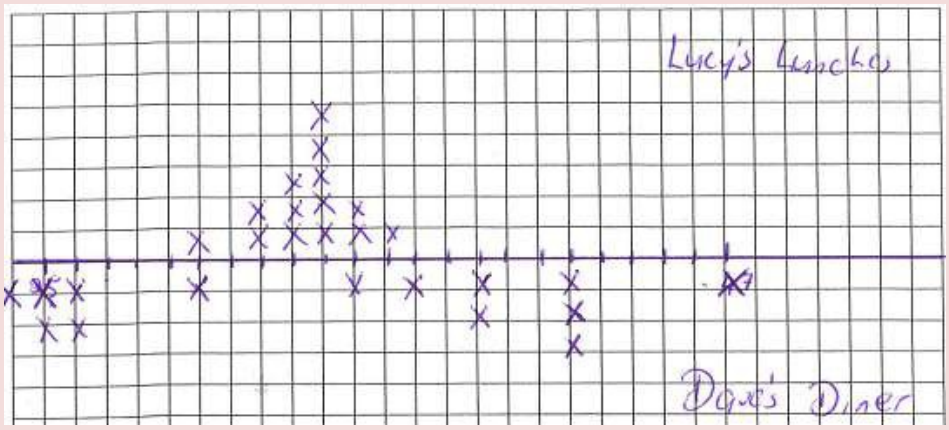
Help Rosin and Peter use their data to decide which restaurant gives the better value for money.

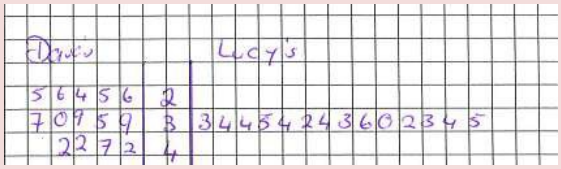
How would **you** investigate which restaurant gives the best value for money?

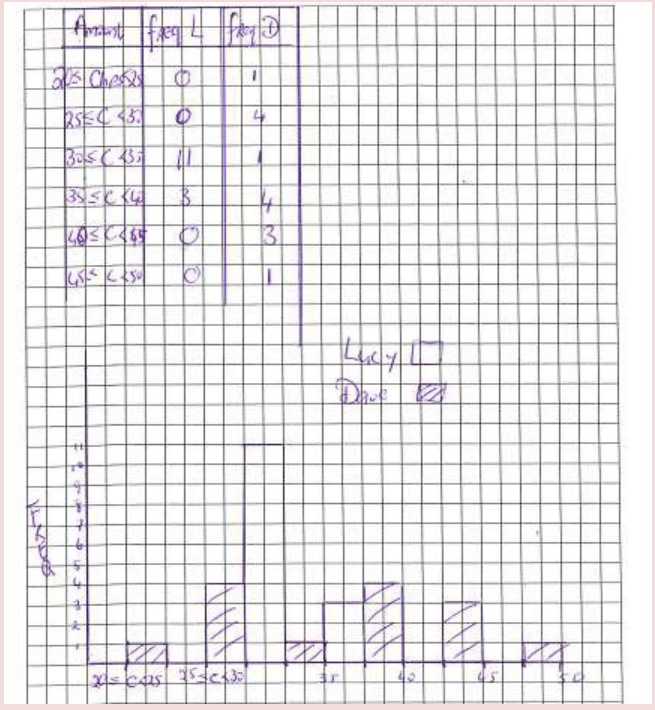
Once you have gathered the data remember you need to display it in a way that allows you to see patterns in the variation.

Think about the different displays you have used throughout the JC course. Think about what makes each of these displays useful.

Look at the displays below that other students made of the data. Which do you think is most useful and why? How would you display this data?





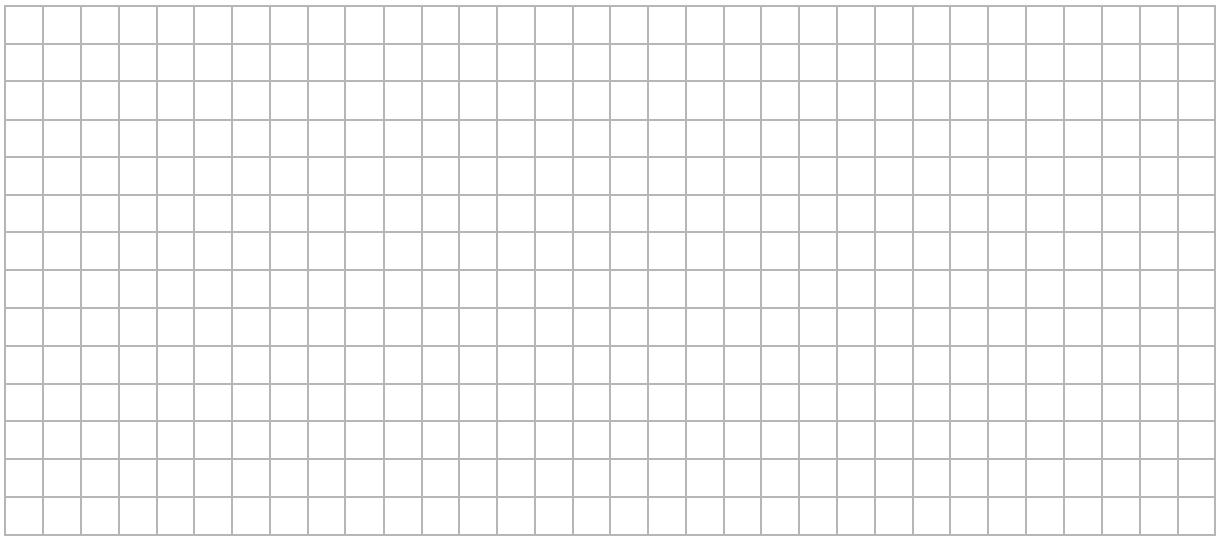


Q. A group of students was investigating the weight of coins. They weighed a **sample** of 48 *two-cent coins* and recorded the weights to the nearest .01g in the table.

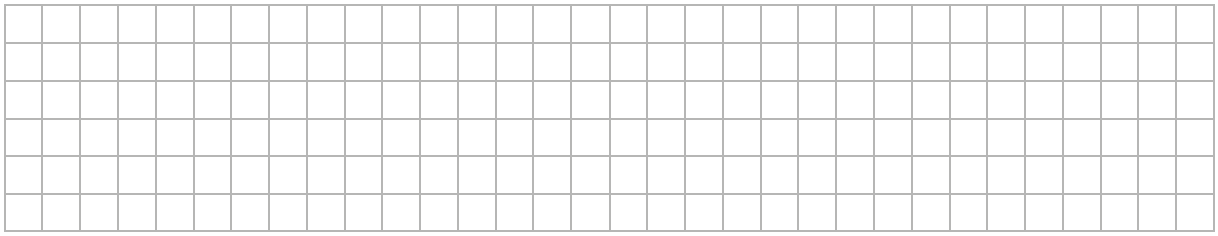
Weight of a two-cent coin (g)							
3.08	2.95	3.10	2.97	3.02	3.10	2.84	3.00
3.10	3.12	3.03	2.85	3.09	3.05	3.15	3.09
3.06	3.05	3.11	3.07	3.02	3.05	3.06	3.18
3.05	3.14	4.52	3.43	3.00	3.09	3.07	2.94
3.05	3.15	3.15	3.00	3.04	3.07	3.06	3.17
3.13	3.05	3.11	3.12	3.03	3.09	3.00	3.01

a) None of the 48 coins weighs the same. What do you think may be a cause of this variation?

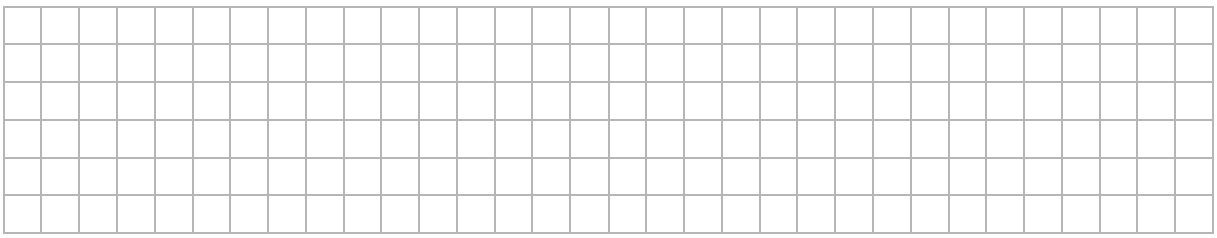
b) Display the data in a way that will allow you to describe it. What do you think is a typical weight for a two-cent coin? Explain your reasoning.



c) Based on the data in the table what do you think the weight of a 49^{th} two-cent coin will be? Are you more confident to give an actual value or a range of values? Explain your thinking.



d) Are there any unexpected values in this data set? How do you know?



Q. Sarah, Ellie and Samir were measuring the length of the science lab. Sarah used a **metre stick**. Ellie and Samir used a **measuring tape**.

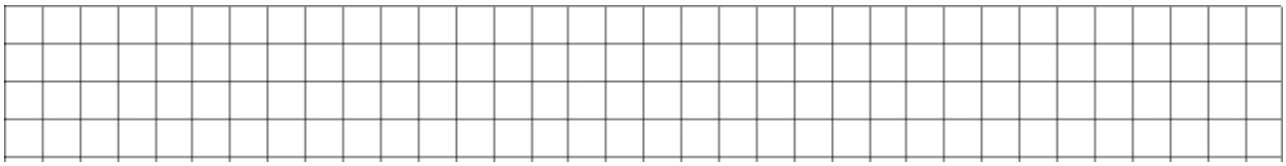
Each group of students measured the length of the lab 6 times and recorded the measurements to the nearest cm in a table

Well each time I worked out how many paces it took for me to walk down the lab. Then I measured the length of a pace with the metre stick and multiplied that by the number of paces and wrote it in the table.

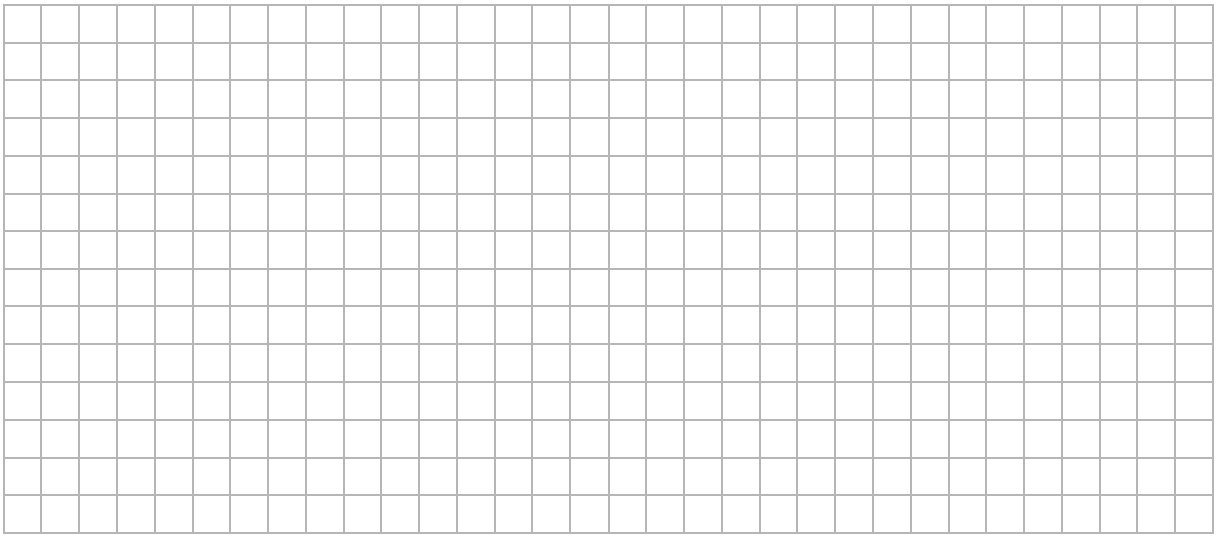
Samir and I worked together. He held the tape against the wall and I walked to the opposite wall and read the measurements. Then we changed, I stayed at the wall and Samir walked down and took the reading; we measured it 6 times.

Measuring Instrument	Length of lab (cm)					
Metre stick	850	870	910	880	915	885
Measuring tape	889	888	889	889	888	888

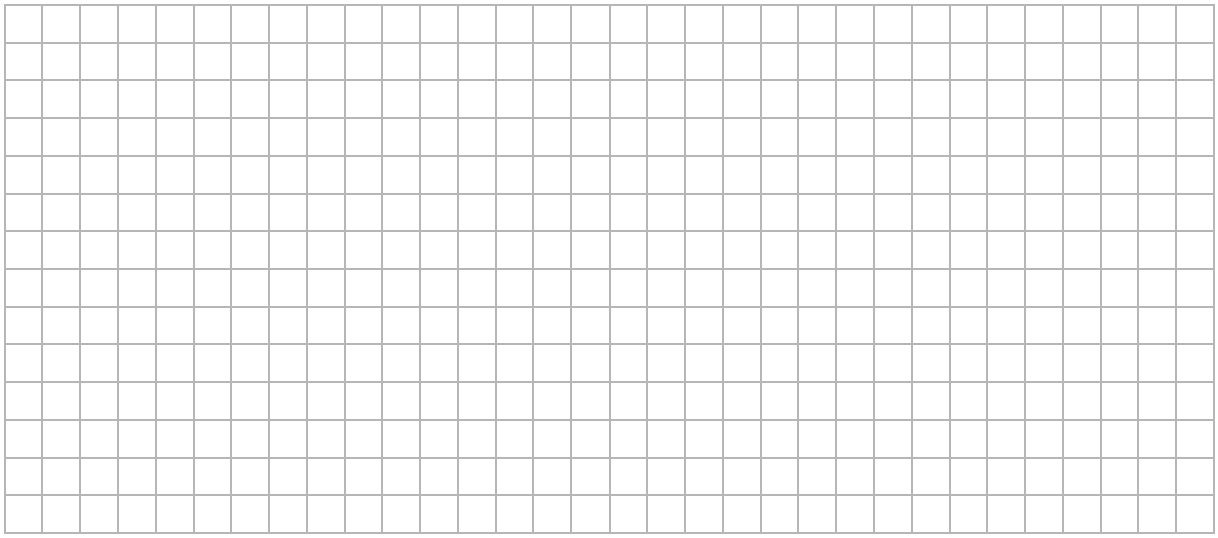
a) Why do you think there are differences in the measurements in the table?



b) Which method gave a more accurate measurement of the length of the science lab? Explain why you think this is the case.



c) If you were asked to state the length of that science lab, what answer would you give? Explain why.



Q Esperanza was investigating family sizes.

She wanted to find out what was a typical family size for people in her class

She asked four classmates:

How many people in your family?

1

Just me and Mum

Sarah

11

Amy

Well my Dad lives with Mary and her 3 children…..that's 5
Karl and I live with Mum, Joe and his daughter Sue..that's another 5….. Oh and Jake the dog.

Mum, Dad Sam and I

8

Paul

Nana and granddad Jones, My other nana, Mum, Dad, me and Jess…Oh and uncle Sean sometimes

4

Ellie

Esperanza's data is not **reliable**? Explain why

If you wanted to find out the typical family size of people in **your** class what would **you** do to make sure the data you gathered was **reliable**?

Below is such data gathered by a group of 3rd Year students.

3,3,3,4,4,4,4,4,5,5,5,6,7,10,4,2,2,5,5,8,4,3,3,3,4,4,4,3,3,3

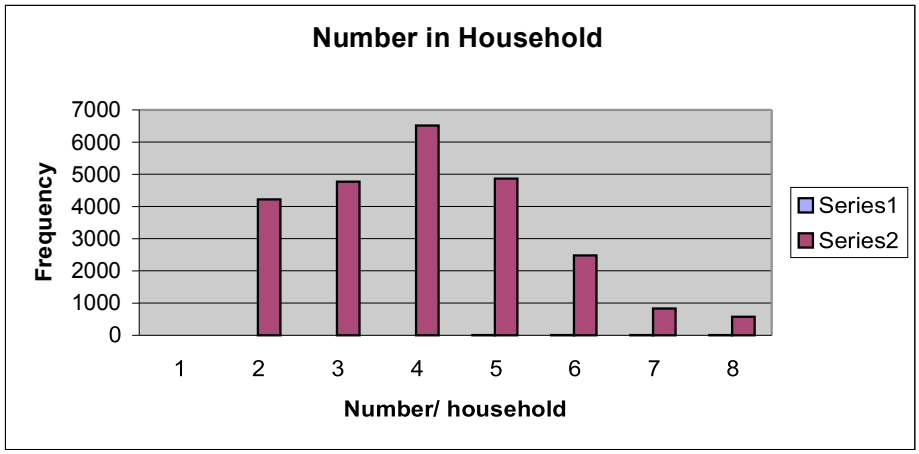
Display this data in a way that allows you to see a pattern in the data.

What would you say is the typical family size for students from this 3rd year group?

Justify your answer with evidence from the data.

The diagram shows the distribution of household sizes from households in the Carlow area.

Household size	Frequency
2	4218
3	4773
4	6512
5	4870
6	2478
7	833
8	572



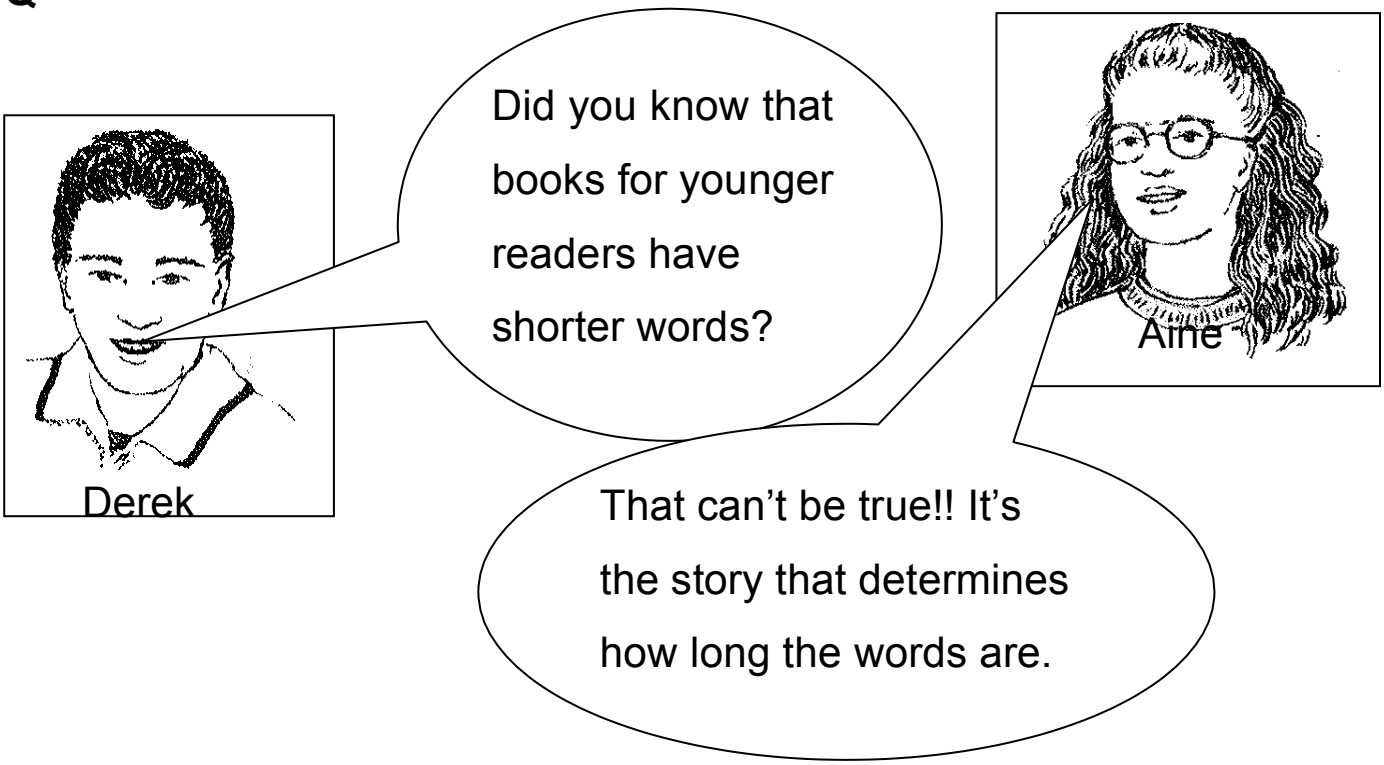
Compare the set of data from Carlow with the 3rd year set.

How likely is it that the 3rd year group surveyed all live in Carlow?

Justify your answer with evidence from the data.

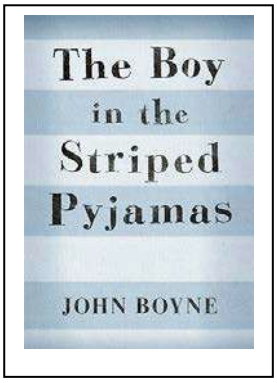
I think it is (impossible, unlikely, likely, certain) that the 3rd year group lived in Carlow because

Q



Use the following extracts from ***Charlie and the chocolate factory*** and ***The boy in the striped pyjamas*** to see if there is any evidence that the words in ***The boy in the striped pyjamas*** are longer than the words in the ***Charlie and the chocolate factory.***

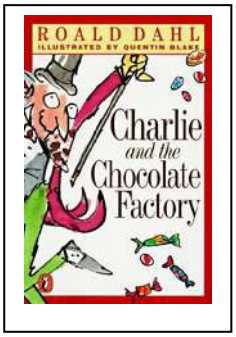
Extract 1



The boy in the striped pyjamas

…One afternoon, when Bruno came home from school, he was surprised to find Maria, the family's maid — who always kept her head bowed and never looked up from the carpet — standing in his bedroom, pulling all his belongings out of the wardrobe and packing them in four large wooden crates, even the things he'd hidden at the back that belonged to him and were nobody else's business. ….

Extract 2



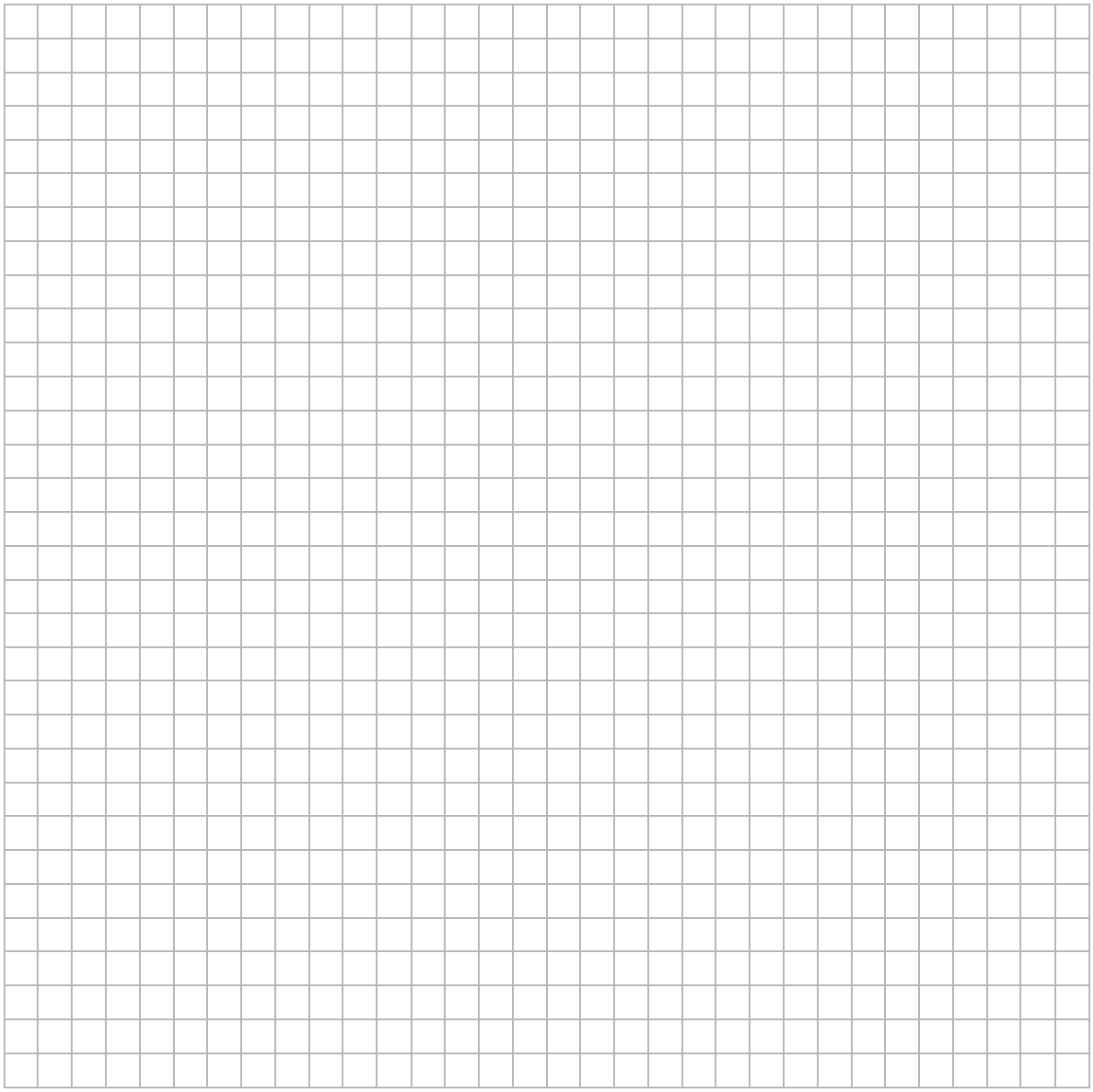
Charlie and the chocolate factory

….. he did. He told all the workers that he was sorry, but they would have to go home. Then, he shut the main gates and fastened them with a chain. And suddenly, Wonka's giant chocolate factory became silent and deserted. The chimneys stopped smoking, the machines stopped whirring and from then on, not a single chocolate or sweet was made. Not a soul went in or out…..

What would **you** do differently if you were going to look for evidence to support Derek's theory?

Think about

- how you would select your sample of words from both books
- the size of your sample.



How does the display help you decide on the *typical* value?

Do the different contexts make it easier or more difficult to state the *typical* value?

How do the *mean*, *mode*, *median* and *range* relate to the *typical* value?

Q. Samil drops a tray with these objects on it

They fall on a wooden floor

How likely are they to break?

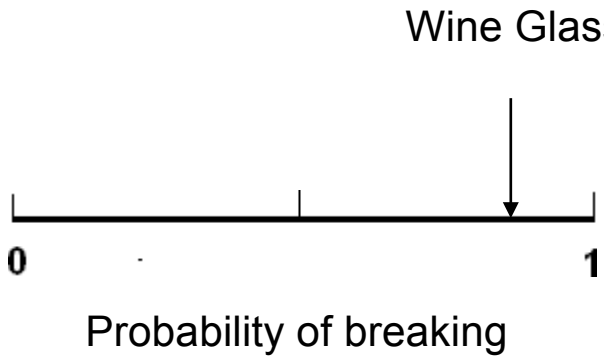
Put them all in order

Most Likely ……

……

……

Least likely ……



The arrow on the number line shows the probability of the wine glass breaking

Explain why this is a sensible place to put the arrow.

Put more arrows on the line to show the probability of the other objects breaking.

Think about the probability of the calculator breaking.

Roughly how far along the line did you put its arrow?

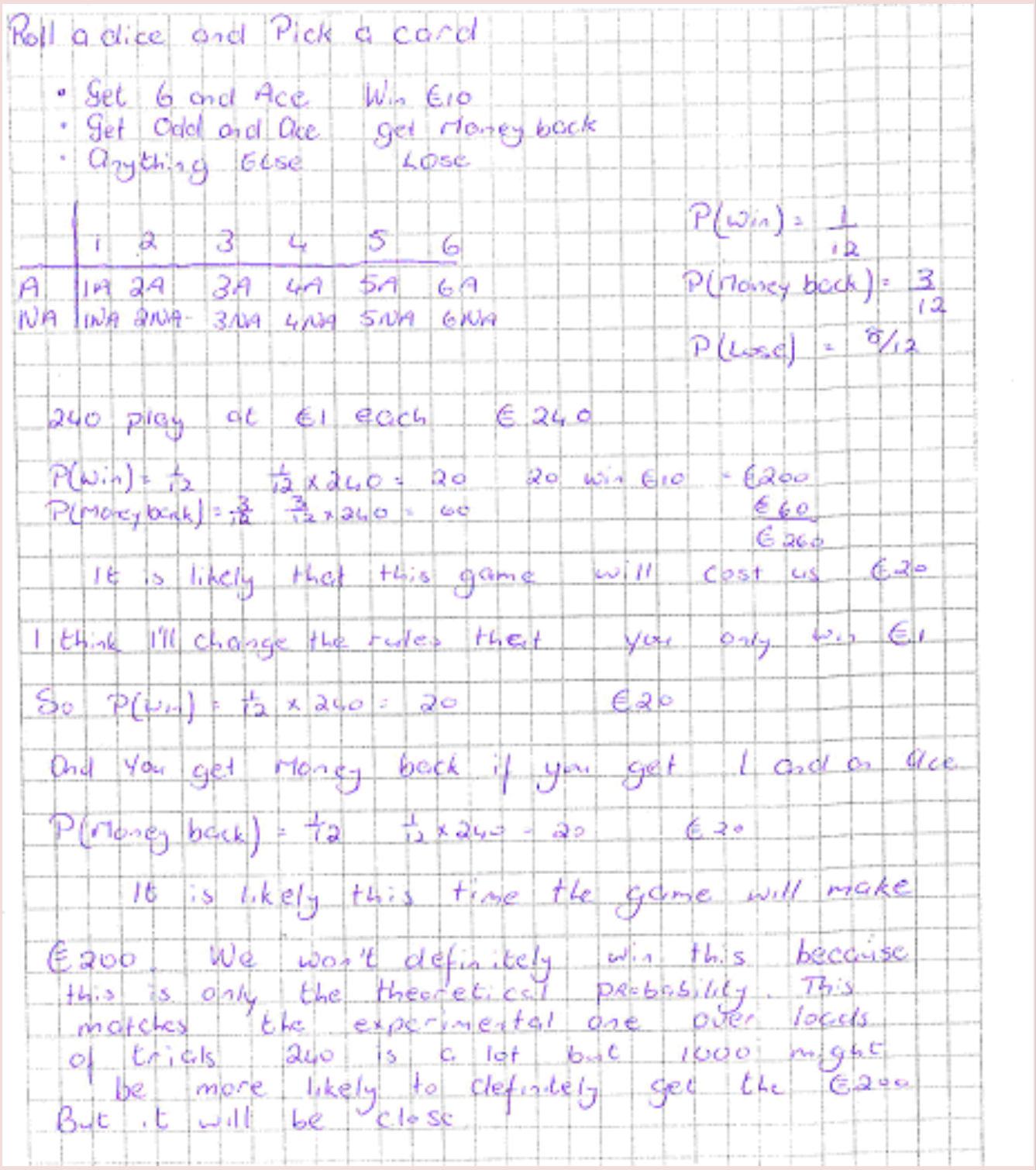
Write this as a **decimal,** a **percentage** or a **fraction**.

Q Devise a game of chance that can be played in school to raise money for charity.

Your game must involve **two independent events,** for example, ‘tossing two coins’ or ‘rolling a die and tossing a coin’.

- Invent a clear set of rules for your game. You should clearly state the conditions for **winning**, **losing** and getting your **money back**.
- Give an example of how you might **“win”** the game, how you might **“lose”** the game, and how you might just get your “**money back”.**
- Decide on how much you will charge to play the game and how much a player will get if they win the game.
- Create a sample space showing **all** possible outcomes.
- Calculate the probability of winning the game.
- Assuming that 250 students play the game, calculate the profit you are likely to make.
- Will you definitely make this profit? Explain why, or why not.

Examine this piece of student work.



Roll a dice and Pick a card

- Get 6 and Ace Win €10
- Get Odd and Ace get Money back
- Anything Else Lose

	1	2	3	4	5	6
A	1A	2A	3A	4A	5A	6A
NA	1NA	2NA	3NA	4NA	5NA	6NA

$P(Win) = \frac{1}{12}$

$P(Money\ back) = \frac{3}{12}$

$P(Lose) = \frac{8}{12}$

240 play at €1 each €240

$P(Win) = \frac{1}{12}$ $\frac{1}{12} \times 240 = 20$ 20 win €10 = €200

$P(Money\ back) = \frac{3}{12}$ $\frac{3}{12} \times 240 = 60$ €60

€260

It is likely that this game will cost us €20

I think I'll change the rules that you only win €1

So $P(Win) = \frac{1}{12} \times 240 = 20$ €20

And You get Money back if you get 1 and an Ace

$P(Money\ back) = \frac{1}{12}$ $\frac{1}{12} \times 240 = 20$ €20

It is likely this time the game will make €200. We won't definitely win this because this is only the theoretical probability. This matches the experimental one over loads of trials 240 is a lot but 1000 might be more likely to definitely get the €200 But it will be close

What do you think of this piece of work? What would you do differently?

Q. Sarah and Caoimhe were raising money to help buy PPE equipment for the local hospital. They created a game of chance called ***Score 10 to win 10*****.** They charged **€1** to play and the prize for winning was **€10.**

Rules: Spin a spinner numbered 1-4 and throw a die.

- If the total is odd get your money back.
- **Score 10** and Win 10.

Create a sample space showing all the **possible outcomes**
Identify those outcomes that are a “win” and those that will get the money back.

Calculate the probability of a player winning the game.

Calculate the probability of a player getting their money back.

If 240 students play the game, how much will Sarah and Caoimhe raise for the hospital PPE?

Change the rules so as to increase the profits, assuming the same number of students play.

Q. The table below shows the main causes of death in Ireland in the years **2000**, **2001** and **2002.**

Numbers

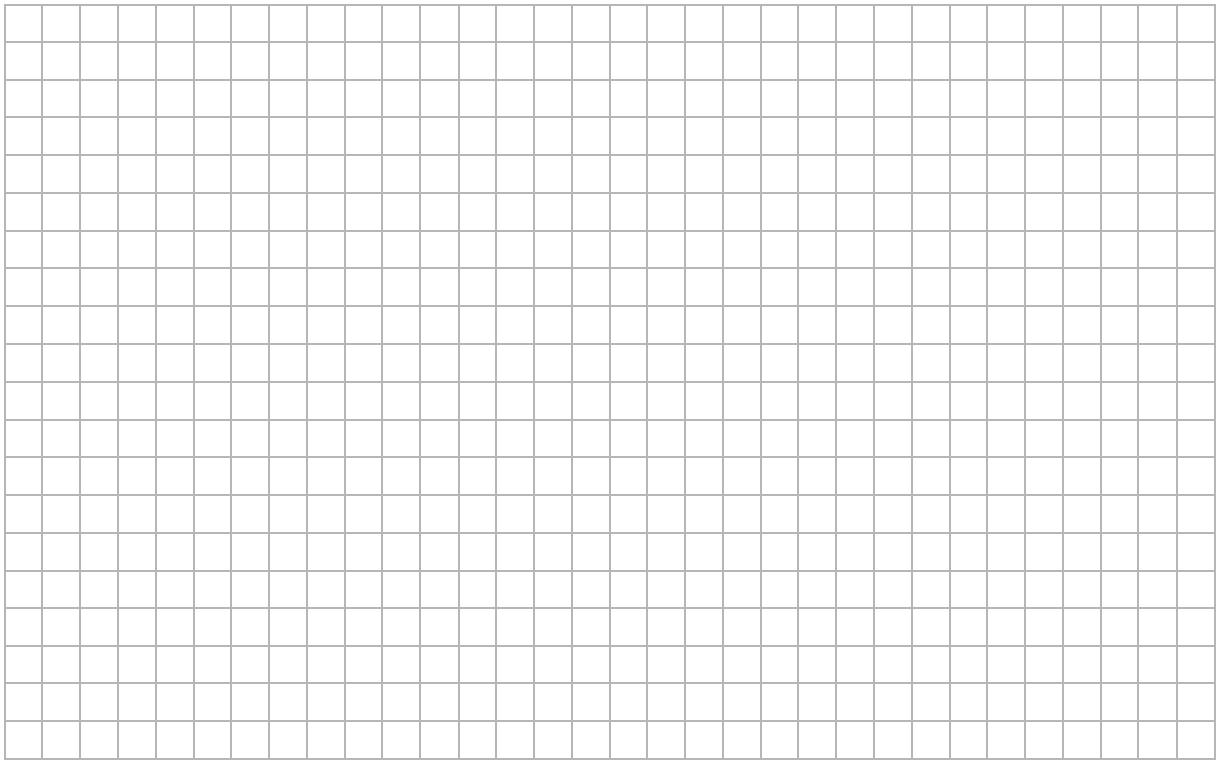
Principal cause	2000 Males	2000 Females	2001 Males	2001 Females	2002 Males	2002 Females
			Republic of Ireland			
Circulatory diseases	6,449	6,217	6,109	5,777	5,886	5,709
Respiratory diseases	2,326	2,537	2,156	2,316	2,118	2,212
Cancer	4,079	3,587	4,038	3,594	4,066	3,433
Road traffic accidents	326	109	316	95	269	85
Suicides	395	91	429	90	371	80
All other	2,617	2,658	2,643	2,649	2,507	2,612
Total deaths	**16,192**	**15,199**	**15,691**	**14,521**	**15,217**	**14,131**

What was the main cause of death of **males** in Ireland in 2002?

Newspaper reports in 2002 stated that the number of **male suicides** was on the increase in Ireland. Is there evidence from the table to support this claim?

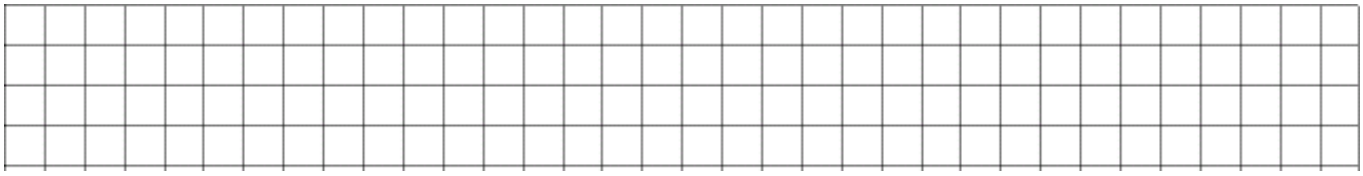
What has happened to the total number of deaths in Ireland in the period from 2000 to 2002?

Newspaper reports claim that more **young Irish males** commit suicide than **young Irish females.** Is there evidence in the table to support this claim?

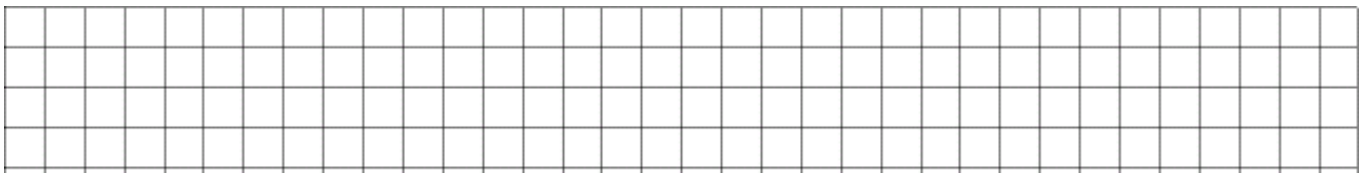


Q. A lottery takes place twice a week in Ireland. **Seven** balls are randomly chosen from **45**. If you match the first six balls drawn then you win or share the jackpot.

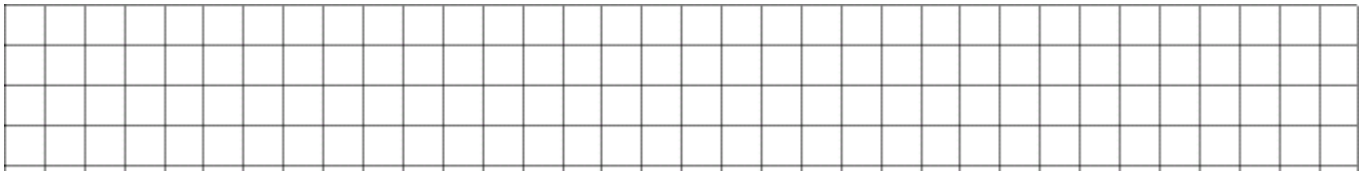
What is the probability of drawing ball number **5**?



What is the probability of drawing an **odd numbered** ball?



What is the probability of drawing an **even numbered** ball?



Below is a table showing the frequency in brackets that the balls have appeared to date.

1 (58)	**2**(69)	**3**(70)	**4**(68)	**5**(68)	**6**(53)	**7**(59)	**8**(53)	**9**(65)
10(63)	**11**(55)	**12**(58)	**13**(74)	**14**(51)	**15**(66)	**16**(48)	**17**(62)	**18**(58)
19(58)	**20**(54)	**21**(69)	**22**(61)	**23**(61)	**24**(64)	**25**(40)	**26**(58)	**27**(67)
28(66)	**29**(59)	**30**(55)	**31**(58)	**32**(68)	**33**(58)	**34**(51)	**35**(60)	**36**(70)
37(55)	**38**(65)	**39**(72)	**40**(57)	**41**(54)	**42**(55)	**43**(39)	**44**(70)	**45**(53)

Write a list of the 5 most frequent numbers and the 5 least frequent numbers.

People often think that ***13*** is an unlucky number and will not choose it. Is there any evidence in the table to suggest that ***13*** is an unlucky number? Explain your reasoning.

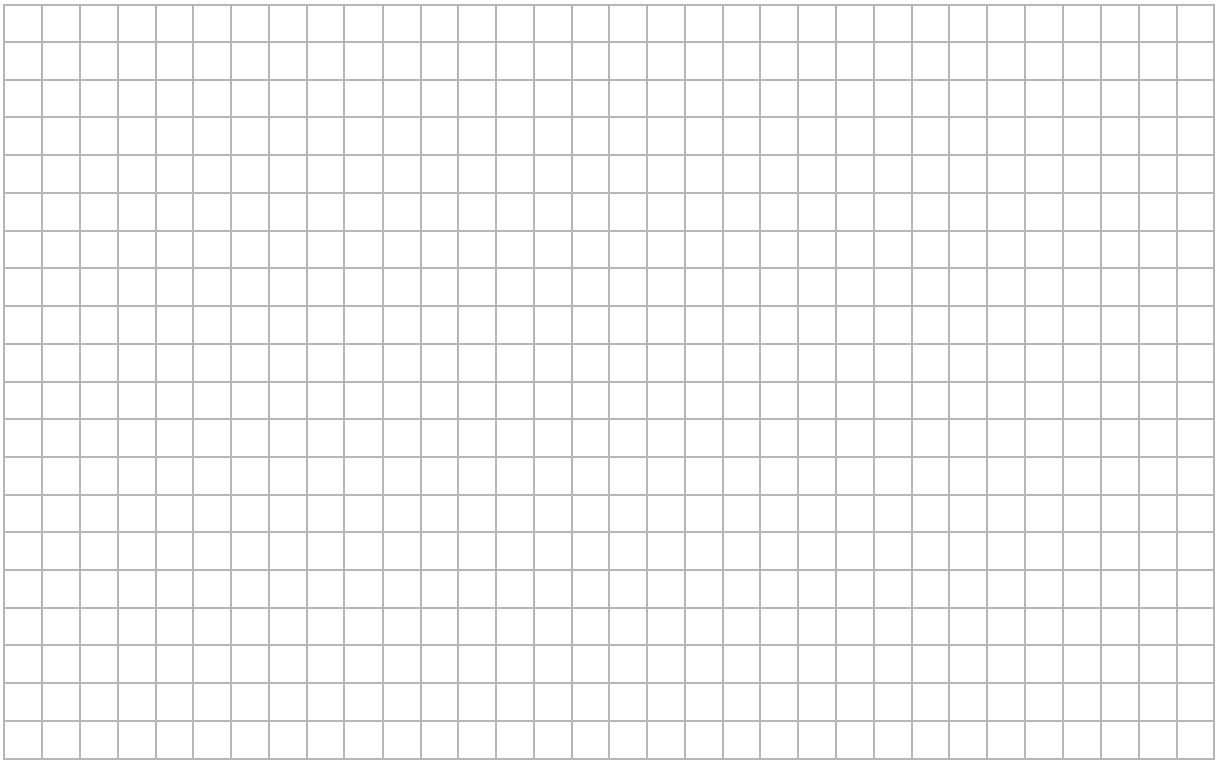
Q. The table shows the total rainfall that fell in Ireland in the **month of July** over a 51 year period from 1958 to 2008.

Year	Total Rainfall (mm)	Year	Total Rainfall (mm)	Year	Total Rainfall (mm)
1958	110	1975	28	1992	69
1959	45	1976	83	1993	60
1960	140	1977	26	1994	65
1961	52	1978	51	1995	70
1962	68	1979	47	1996	37
1963	24	1980	39	1997	54
1964	47	1981	36	1998	54
1965	79	1982	9	1999	35
1966	37	1983	18	2000	44
1967	84	1984	31	2001	30
1968	16	1985	107	2002	68
1969	44	1986	58	2003	46
1970	68	1987	33	2004	38
1971	63	1988	80	2005	84
1972	41	1989	10	2006	18
1973	79	1990	48	2007	119
1974	100	1991	26	2008	112

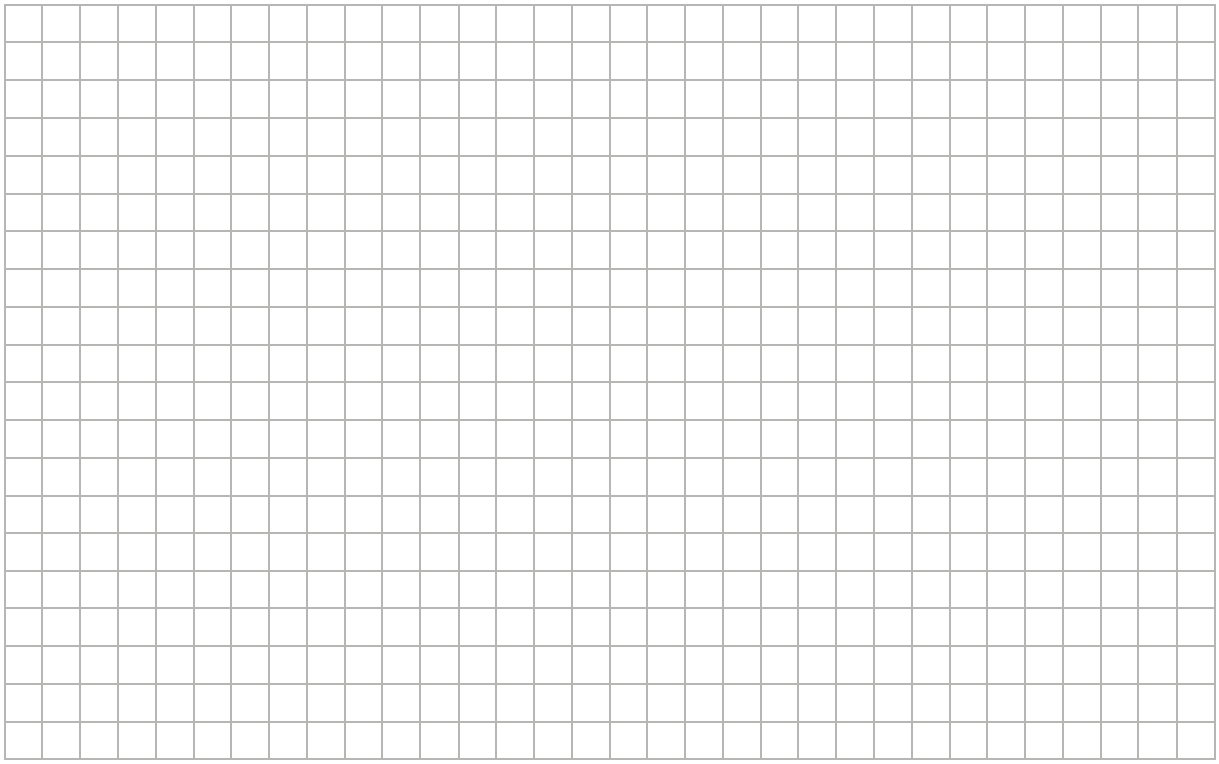
If 130mm of rain fell in Ireland in July 2009, complete the table below showing the total rainfall for each of the decades listed.

Years	Total Rainfall (mm)
1960-1969	
1970-1979	
1980-1989	
1990-1999	
2000-2009	

Display your data in a way that allows you to see a pattern in the variation.



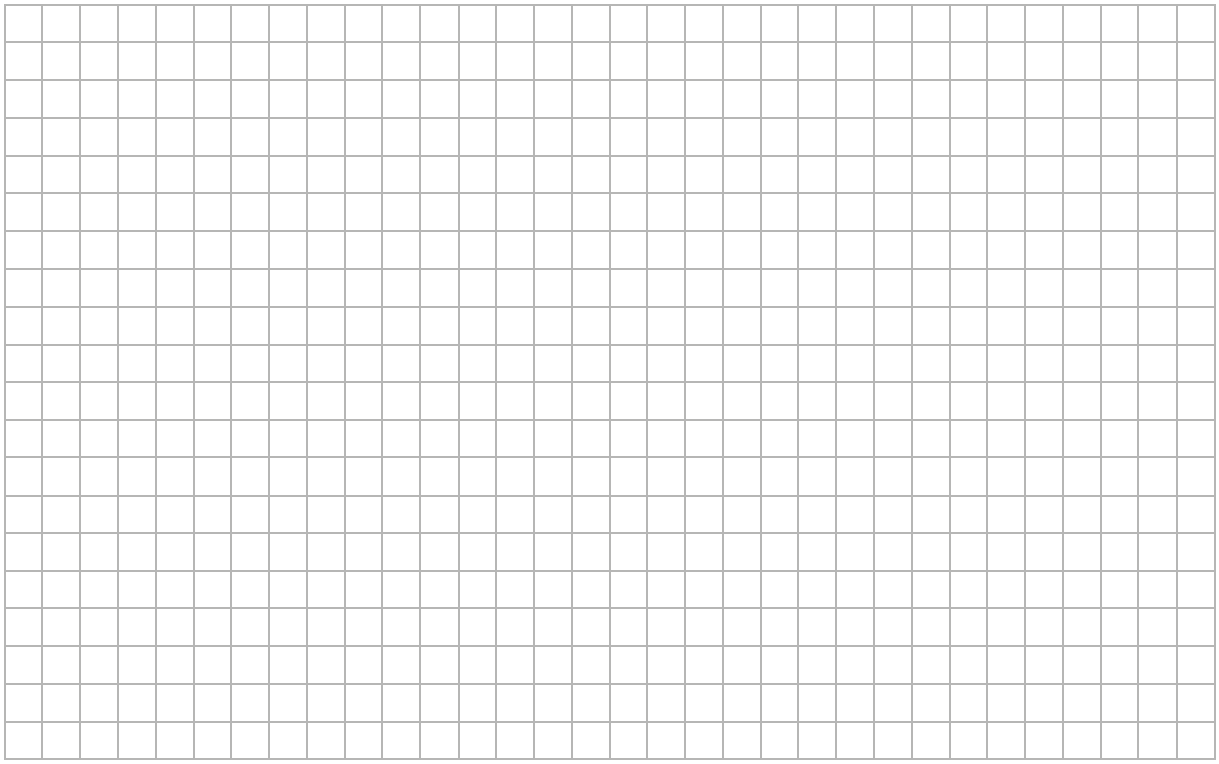
Is there any evidence to support the claim; *Julys in Ireland are getting wetter?*



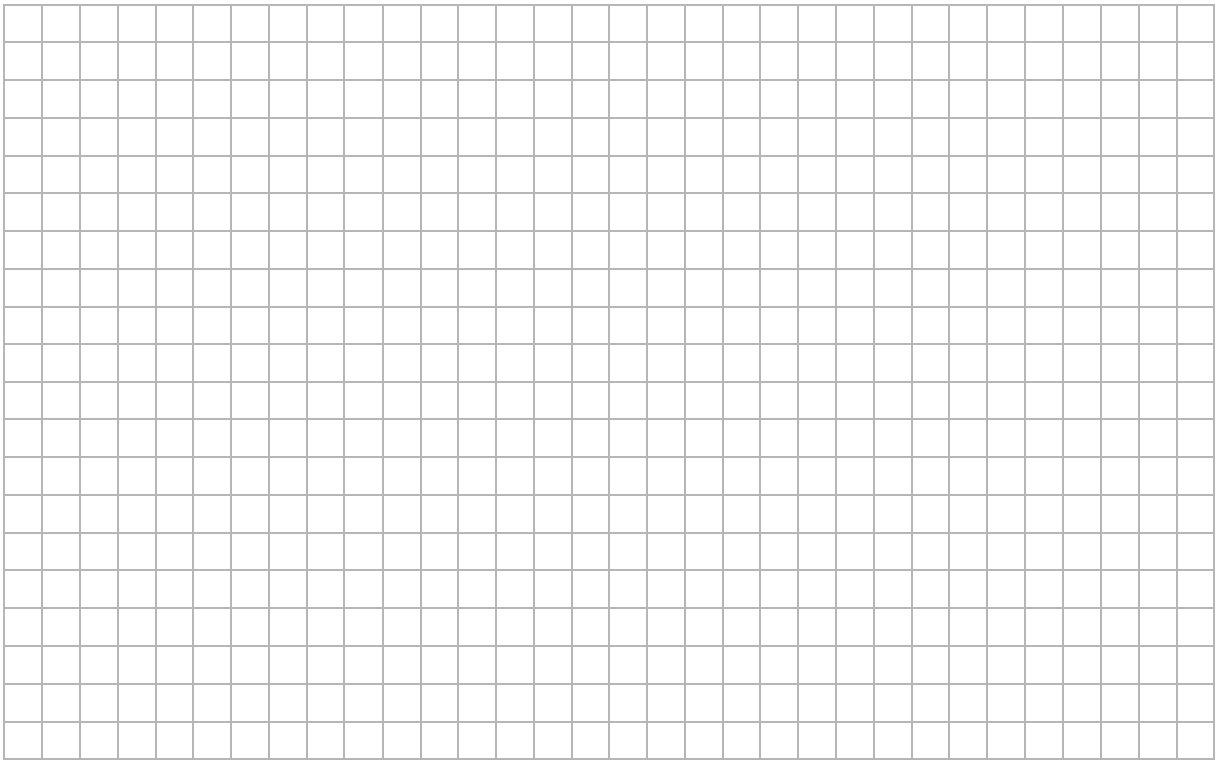
Q. The table shows the number of hours per day spent by 3rd year and TY students playing on a games console.

Number of hours spent playing on a games console	Number of TY Students	Number of 3rd Year Students
1		
2	1	1
3	2	3
4	1	1
5	1	2
6	5	2
7		3
8		
9	1	3
10		1
11		3
12		2
13	3	3
14	1	1
15	4	
16	4	3
17	2	1
18	4	2
19	4	4
20	3	2
21	2	
22	3	
23	1	
24		
25	1	4

Display the data in a way that allows you to compare the two groups.



Which group of students spends more time playing on a games console? Give evidence from the data to support your answer.

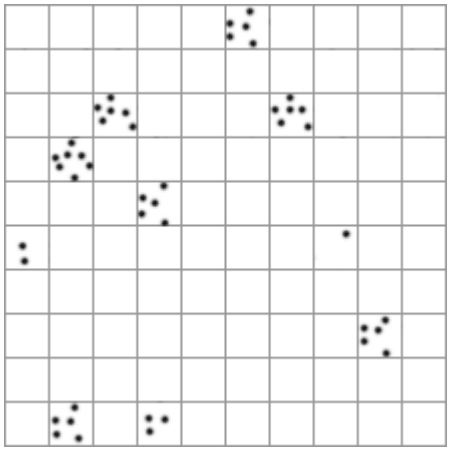


Q. Photographs taken from satellites help officials keep track of the number of different objects on the earth below.

They could be keeping track of the number of animals in remote areas, or counting the number of sheep on a farmer's land to ensure that they are claiming the correct subsidy.

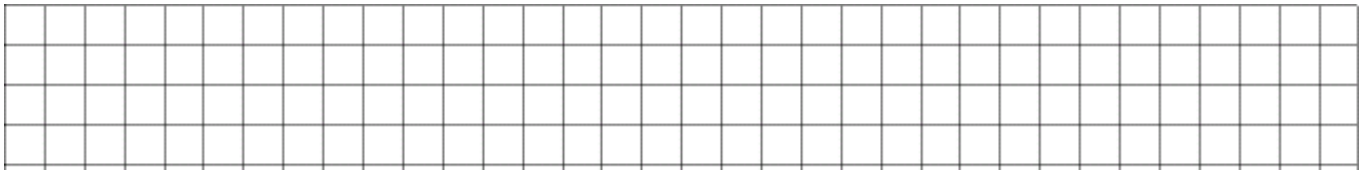
Below is a photograph of a sub-Saharan region in Africa, populated by gazelle. The area is divided into 100 sub-regions. Some of the sub-regions are obscured by cloud.

A gazelle

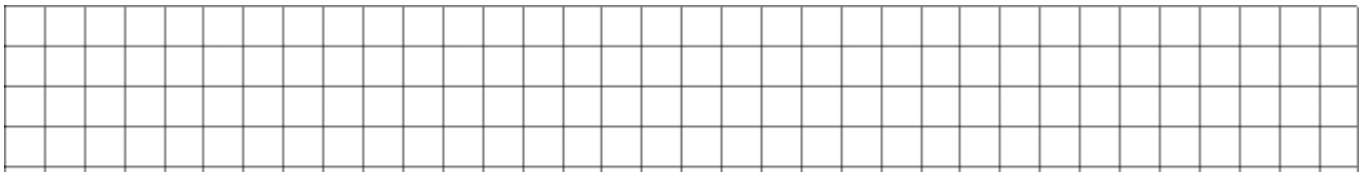


Based on the number of gazelle in this sample, make an estimate of the number of gazelle in the entire region.

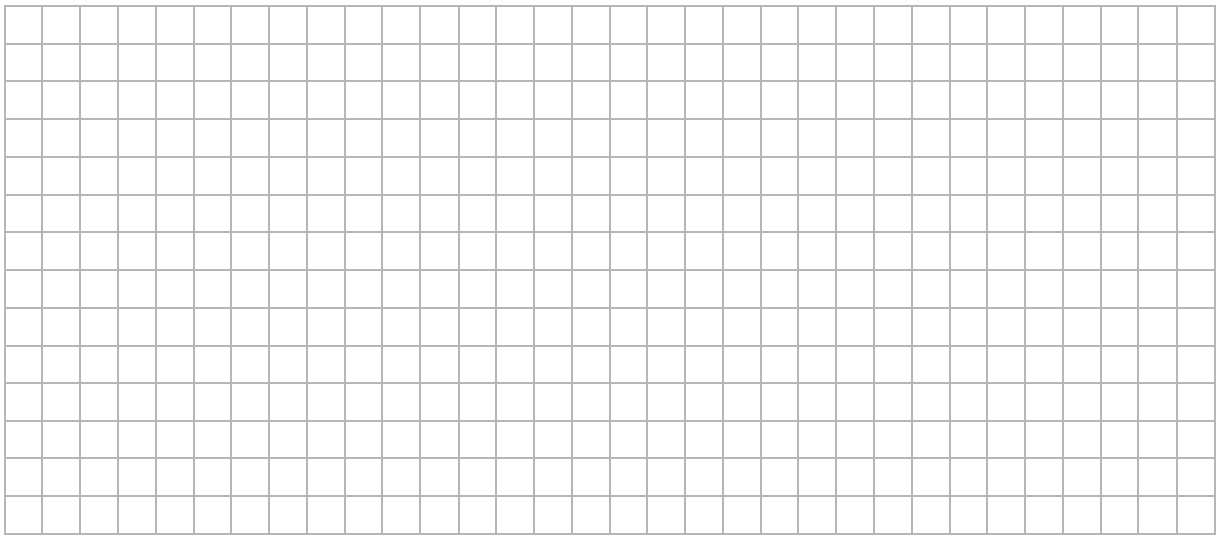
Is it possible that there are more gazelle than your estimate?



Is it possible that there are less gazelle than your estimate?



How might a smaller sample size have affected your estimate? Consider some smaller samples from the photograph to help in explaining your thinking.



Q. The table below reports the approximate lowest frequency and approximate highest frequency of the hearing ranges for humans and five other animals. Frequency is measured in Hertz (Hz), which is 1 vibration per second.

Animal	Lowest frequency	Highest frequency
Human	64	23,000
Cat	45	64,000
Dog	67	45,000
Gerbil	100	60,000
Goldfish	20	3,000
Parakeet	200	8,500

Which animal has the smallest hearing range?

What animals can hear lower frequencies than a parakeet can?

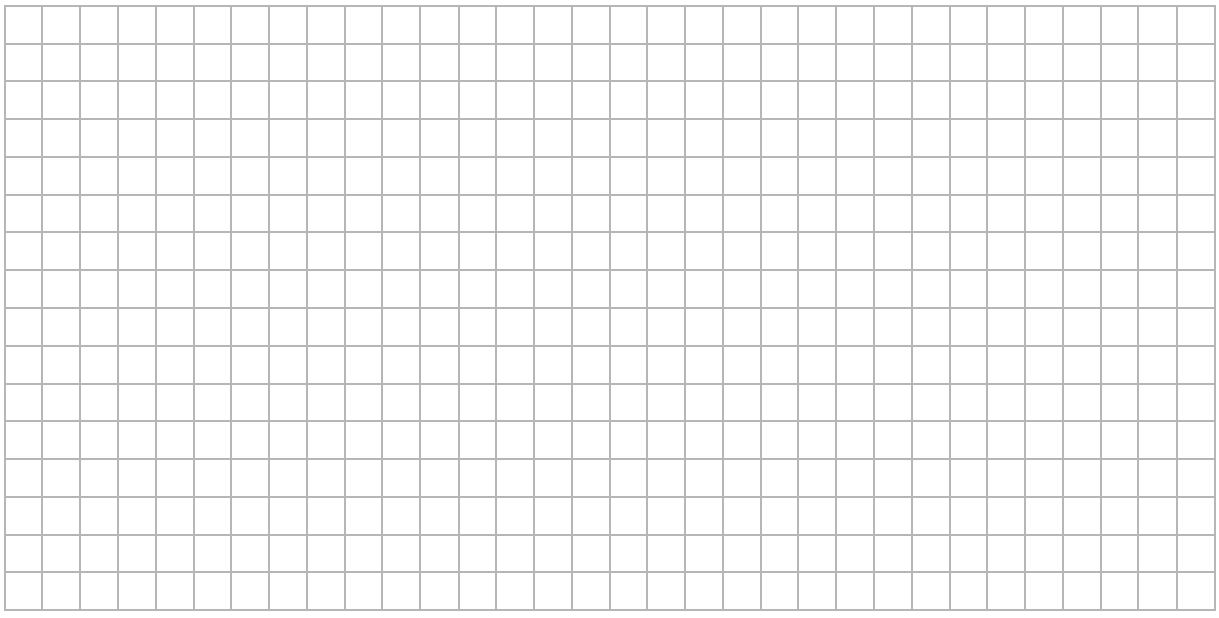
Which animal can hear the highest frequencies?

What animal has a hearing range more than twice that of a human?

Q The table below shows some cloud formations and their recorded distances above the Earth.

Cloud Type	**Distance above the Earth (miles)**
Altocumulus	4
Altostratus	5
Cirrostratus	6
Cirrus	7
Cumulonimbus	2
Cumulus	3
Stratus	1

What is the **median** distance above the Earth of the cloud formations listed above?



Q. Sam asked the 29 students in 3rd year how many times they were absent from school last term. The results are shown in the table below. Unfortunately a blot covers part of the table.

Number of days absent	Frequency
0	3
1	10
2	9
3	
4	
More than 4	1

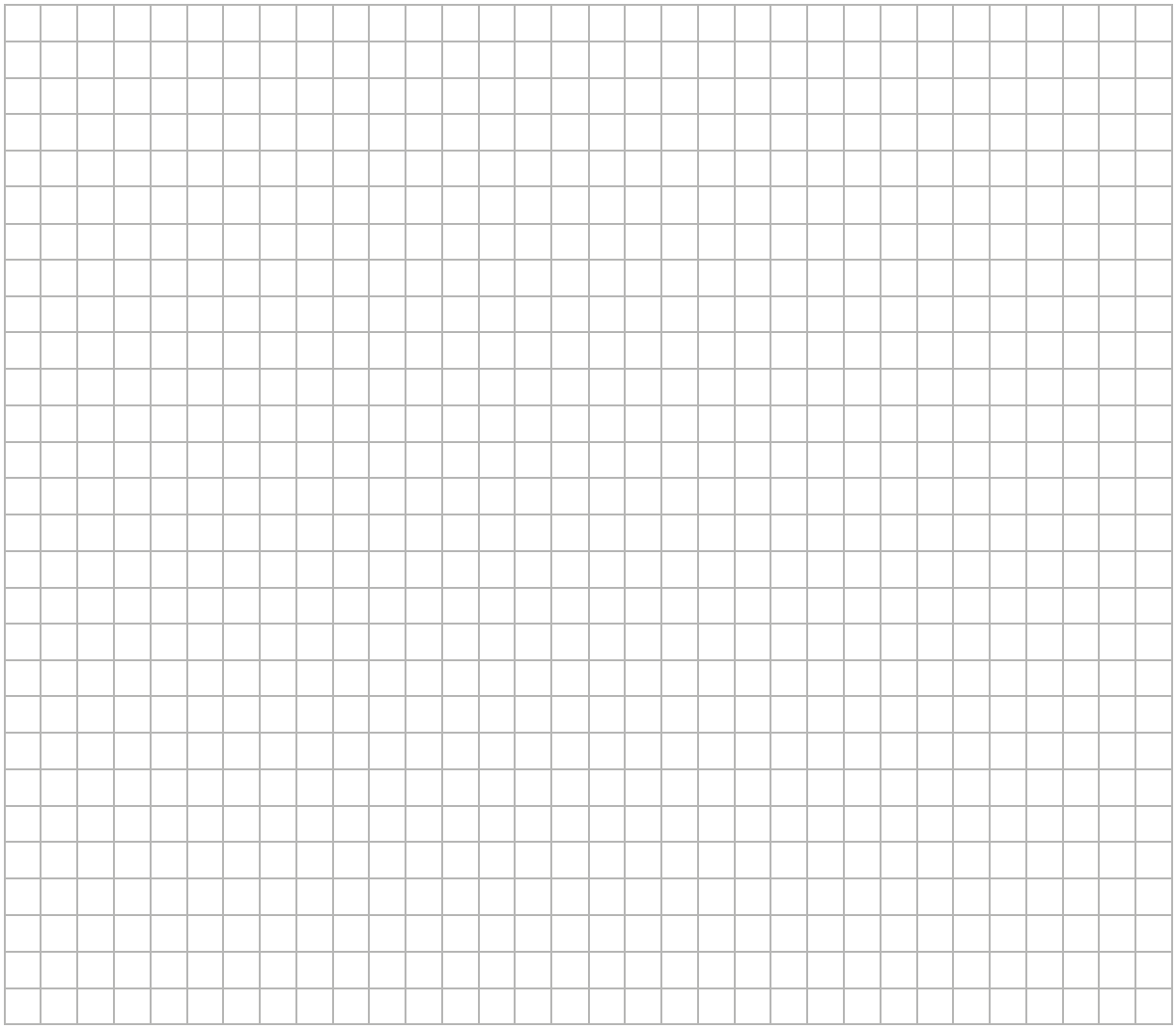
a) (i) What might the table look like if the blot was not there?

Give two possible answers.

No. of days absent	Possible Frequency 1	Possible Frequency 2
0	3	3
1	10	10
2	9	9
3		
4		
More than 4	1	1

(ii) How many possibilities are there, other than the two you have shown?

b) (i) Working from Sam's original table, calculate (if possible) the mode, median, mean and range of the data.



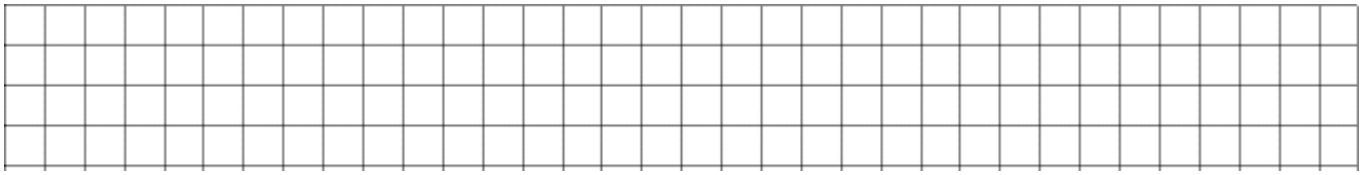
Q. The plot shows the heart rates of a group of 3rd years half way through their PE class.

Heart Rate (Beats/min)

8	7 7 9
9	6 7 7 7
10	4 5 6 6 8
11	1 3 4 9 9
12	2 5 5 8
13	0 0 3 4 7
14	3 5
15	7
16	9

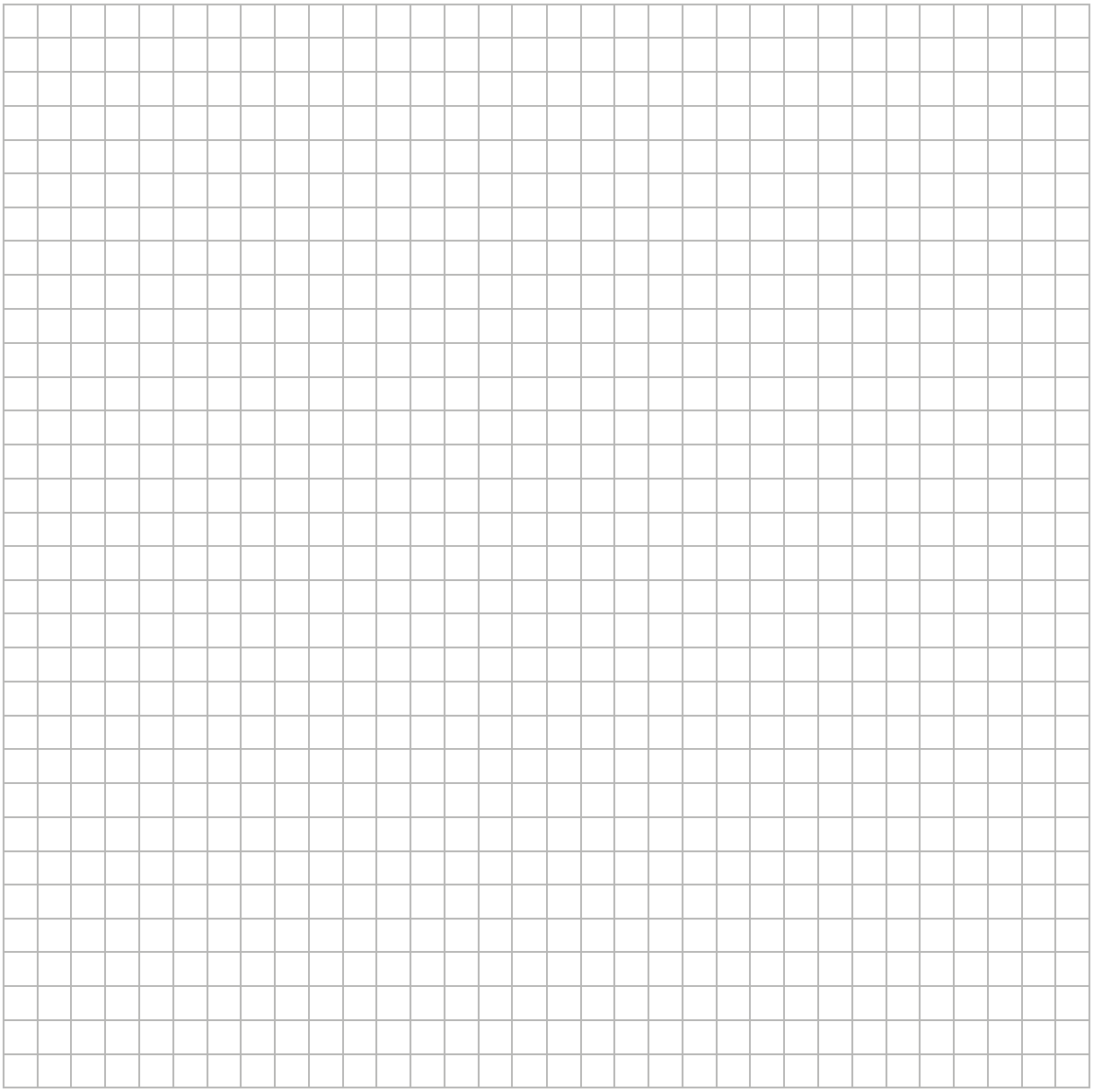
11 | 3 = 113

Optimum heart rate is between 110 and 140 beats per minute.
How many heart rates shown on the plot are between 110 and 140 beats per minute?



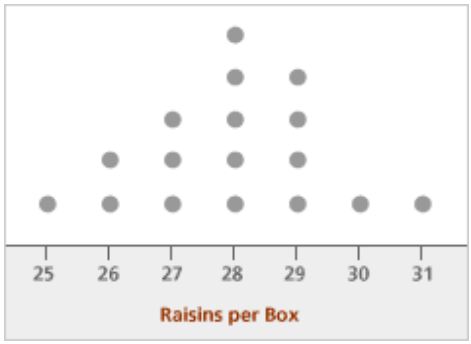
Guidelines state that, if the heart rate exceeds 165 beats per minute, exercise should be stopped immediately.

Should any of these students stop exercising immediately? Explain your answer



Now see how well you have understood these concepts by answering the assessment items that follow. Make a note of the parts of the questions you find difficult or confusing. You may like to discuss these areas with your friends or your teacher.

Q. Students were investigating the number of raisins contained in individual boxes of Sun-Maid raisins. They recorded their results in the diagram shown.



(a) If the students choose a box at random from all the boxes they surveyed what is the probability that the box contains 29 raisins?

(b) Four boxes were found after the students had completed the line plot above.
Jack, Sarah, Amy and Kevin were each given a box and asked to count the contents.
Jack said his contained 28 raisins. Sarah said hers contained 28 raisins also.
Another student said: “I bet Amy’s contains 28 raisins also.
Kevin said “Wait, Amy; don’t reveal the contents of your box yet.”

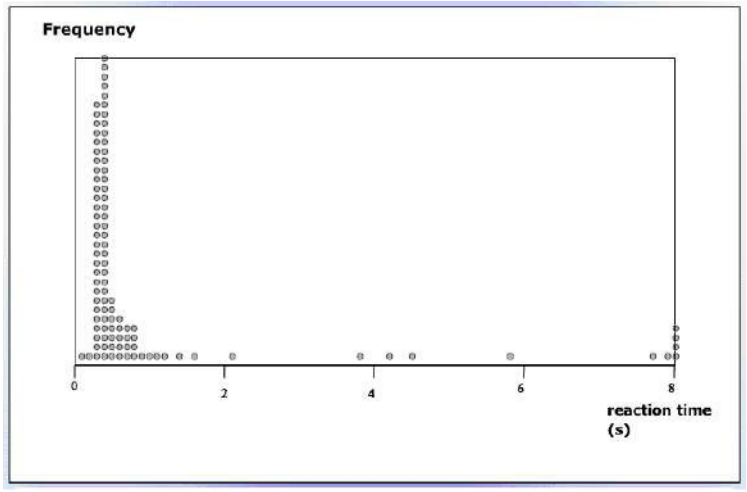
He and Amy whispered together and then Kevin said “I will tell you that if the contents of our two boxes are added to the data the mean number of raisins per box will be 28.

Give one possible value each for the number of raisins in Kevin’s and Amy’s boxes.
Justify your choice.

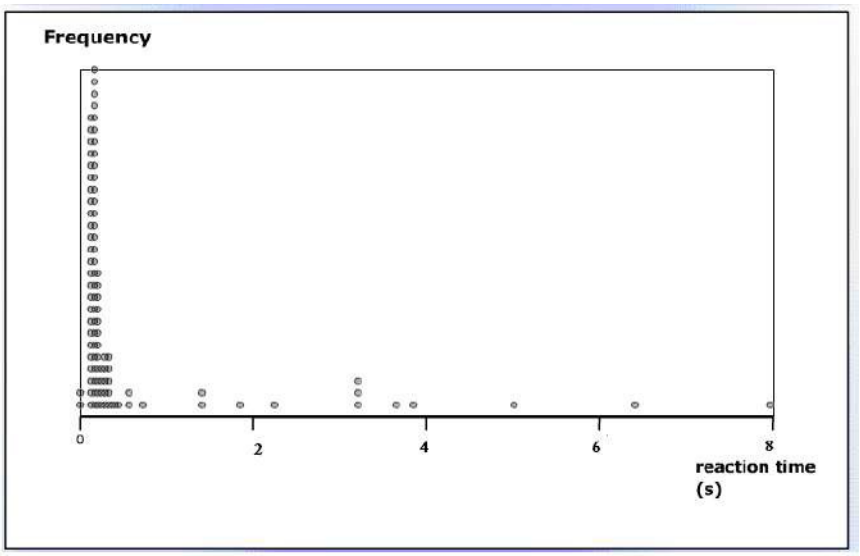
Is it possible that the student won the bet?
Explain your reasoning

Question **(Suggested maximum time: 10 minutes)**

(a) 100 students taking part in a *Censusatschool* survey had their reaction times for a task checked online. The results are shown below.

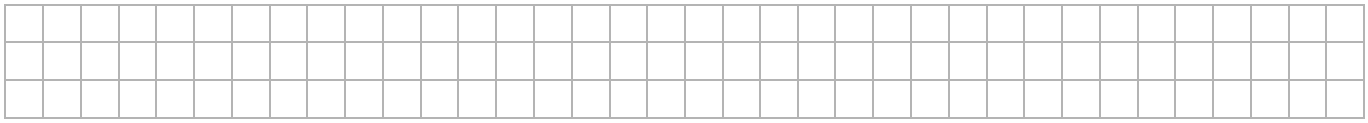


The students were allowed to practise the task and their reaction times were tested again; the results are shown below.



Do you think that this evidence suggests that reaction times can improve with practice? Explain your thinking.

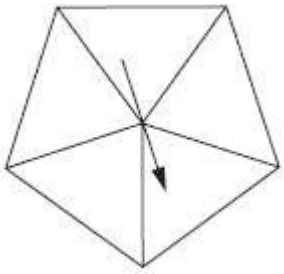
4



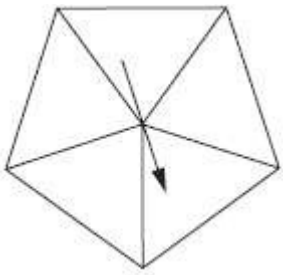
Question **(suggested maximum time: 8 minutes)**

On each spinner write five numbers to make the statements correct.

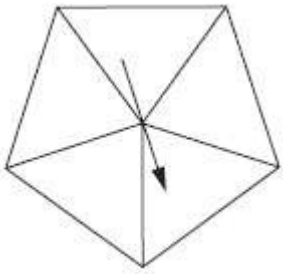
(i) It is *certain* that you will get a number less than 6.



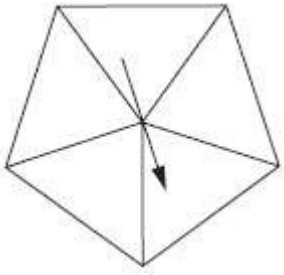
(ii) It is *more likely* that you will get an even number than an odd number.



(iii) It is *impossible* that you will get a multiple of 2.

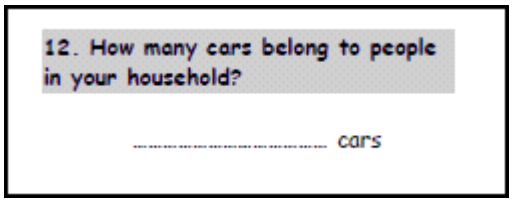


(iv) It is *likely* you will get a prime number.



Question **(Suggested maximum time: 12 minutes)**

The following question was asked on the phase 10 *Censusatschool* questionnaire.



The data below are from groups of students chosen at random from Ireland and South Africa.

No of Cars per Household	
Ireland	South Africa
1	1
1	2
2	0
1	0
1	2
2	0
2	0
2	1
3	1
1	1
1	1
3	1
2	3
5	2
1	2
3	2
6	1
5	1
2	1
3	1
2	1
1	3
2	3
1	2
1	1
1	0
2	1
2	1
1	1
2	1

(a) Display the data in a way that allows you to compare the two groups.

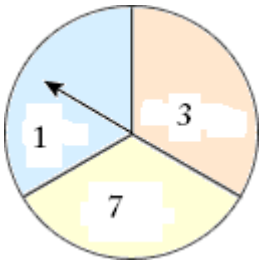
(b) What do you notice about these two groups of students? Is there any evidence that households in one country have more cars than the other?.Explain your answer.

7

Question **(Suggested maximum time 25 mins)**

Sophie and Amy were designing a game of chance to raise money for charity in their school. They agreed on the following:

- they would call the game ***Spin and Win***
- they would charge 50 cent to play
- the rule would be: roll a die and spin the spinner shown and add the totals**.**



(a) Create a *sample space* showing all the possible outcomes.

However, Sophie and Amy had different ideas about which outcomes would result in a win, a loss or the player getting their money back.

Sophie's Idea

Money back: Get total of 13
Win €1: Even number total
Lose: Anything else

Amy's Idea

Money back: Even number total
Win €1: Odd total, but not Prime
Lose: Anything else

(b) What is the probability of winning if you play using Sophie's idea?

(c) What is the probability of getting your money back if you play using Amy's idea?

(d) If you play using Amy's idea, is the probability of winning **greater than** or **less than** the probability of winning if you play using Sophie's idea? Explain your reasoning.

(e) If 180 people play the game using Sophie's idea, how much are they likely to raise for charity? Show how you worked out your answer.

Question

Oxygen levels in a polluted river were measured at randomly selected locations before and after a clean-up. These results were given in the table:

Before (mg/l)				After (mg/l)			
20 23 2	25 23 10	20 10 11	9 11 5 11	26 11 3	10 15 8	10 11 11	9 11 4 13

(a) Construct a back-to-back stem-and-leaf plot of the above data.

(b) State **one difference** and **one similarity** between the distributions of the measurements before and after cleanup.

Difference:

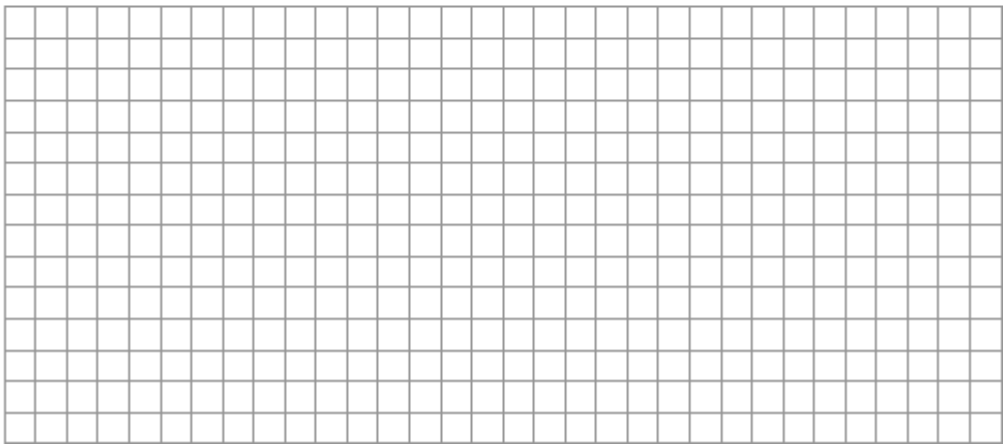
Similarity:

Question 4 **(25 marks)**

Sam wanted to see how well the people in his class could judge how long one minute is.
He asked each student to say *'Start'* and then to say *'Stop'* when they thought a minute had gone by.
Sam used a stop-watch to time each of them.
He recorded the results in the table below.

Times in seconds					
63	56	86	52	75	65
57	59	64	55	89	54
39	67	82	70	68	57
66	72	33	42	52	79
60	59				

(a) Display the data in a stem and leaf plot.



(b) How many people did Sam time?

(c) What is the median time?

Extension

Compare Sam's set of data with the previous set.

Use two **stem and leaf plots** or **back-to-back stem and leaf plots.**

Use % and fractions.

Mention areas where the data is clumped.

State the range of both sets.

Which group do you think was better at estimating a minute?

Justify your answer.

Oxygen levels in a polluted river were measured at randomly selected locations before and after a clean-up. These results are given in the table.

Before (mg/l)				After (mg/l)			
20	25	20	9	26	10	10	9
23	23	10	11	11	15	11	11
2	10	11	5	3	8	11	4
11				13			

(a) Construct a back-to-back stem-and-leaf plot of the above data.

(b) State **one difference** and **one similarity** between the distributions of the measurements before and after cleanup.

Difference:

Similarity:

Some research was carried out into the participation of girls and boys in sport. The researchers selected a simple random sample of fifty male and fifty female teenagers enrolled in GAA clubs in the greater Cork area. They asked the teenagers the question: *How many sports do you play?*

The data collected were as follows:

Boys	Girls
0, 4, 5, 1, 4, 1, 3, 3, 3, 1,	3, 3, 3, 1, 1, 3, 3, 1, 3, 3,
1, 2, 2, 2, 5, 3, 3, 4, 1, 2,	2, 2, 4, 4, 4, 5, 5, 2, 2, 3,
2, 2, 2, 3, 3, 3, 4, 5, 1, 1,	3, 3, 4, 1, 6, 2, 3, 3, 3, 4,
1, 1, 1, 2, 2, 2, 2, 2, 3, 3,	4, 5, 3, 4, 3, 3, 3, 4, 4, 3,
3, 3, 3, 3, 3, 3, 3, 3, 3, 3	1, 1, 3, 2, 1, 3, 1, 3, 1, 3

(a) Display the data in a way that gives a picture of each distribution.

(b) State **one difference** and **one similarity** between the distributions of the two samples.

Difference: _____

Similarity: _____

(c) Do you think that there is evidence that there are differences between the two populations? Explain your answer.

Note: you are not required to conduct a formal hypothesis test.

Answer: _____

Justification:

(a) A teacher asked the students in her class to estimate the height of the church opposite the school in metres.

The stem-and-leaf diagram shows all the results:

3	5	9				
4	0	2	6	8	8	
5	3	3	5	7	7	9
6	0	5	5	5		
7	4	8				
8	2	7				

Key: 3 | 5 represents 35 m

(i) How many students were in the class?

(ii) Describe the **shape** of the distribution of the data.

(iii) What was the median estimate?

(iv) Explain the answer to part **(iii)** to someone who does not know what the word "median" means.

Q. David noticed that, when he drank a bottle of sports drink before going out for a run one day, his performance time improved. He set about doing an experiment to see whether drinking the sports drink increases performance when running.

He recorded the times of people in his running club to complete a 5km run without drinking the sports drink and then on another day he recorded the time it took the same people to complete 5km having taken the sports drink.

He recorded the information in a back-to-back stem and leaf plot:

Without taking the sports drink.		**Having taken the sports drink**.
5	20	3 4
1 1 1	21	3 4 7
8 8 4 3 2	22	
	23	1 2 2
	24	0
	25	8
	26	1
	27	
	28	2 3 6 6 7 7
	29	2 4 4 5 5 5 8 9 9
	30	1 3 4 5 6 7 8 8 9
5	31	
6 4 4 3 0 0	32	1 1 4 9
9 9 6 5 4 4 3 3 2 1	33	3 3 3 2
7 7 5 5 6 6 6 1 0	34	5
8 8 8 3 3	35	0 0
7 3 2	36	1
1	37	2
	38	3 5
2 2	39	
4 4 2 0	40	

Key: 32 **|** 1 means 32.1 minutes

(i) Based on the diagrams approximate the median speed without drinking the sports drink and the median speed having taken the sports drink. What does this information tell you?

(ii) Compare the distributions of each of the data sets above.

(iii) Is there evidence from the diagram to suggest that taking the sports drink improves performance? Justify your conclusions.

(iv) Make an argument, based on the two data sets, that taking the sports drink does not improve performance.

(v) After completing the experiment, David wondered how accurate his study was. He realised that he had not specified how much of the sports drink the runners should take. He asked 20 of the runners approximately how many mililitres of sports drink they had taken and recorded this alongside their time. The results are as follows:

Time (mins)	Sports drink (ml)
20.3	250
21.7	100
21.8	120
24	80
28.6	300
29.4	130
29.5	300
29.9	280
32.1	300
32.1	100
33.2	80
35	220
38.3	180
20.6	100
29.2	200
29.8	250
36.1	80
29.9	120
30.9	240
30.1	280

Display the data in a way that allows you to examine the relationship between the two data sets.

(vi) Is there evidence to suggest that there is a relationship between the time taken to complete 5km and the amount of sports drink taken before the race?

7

Q.A group of students were asked "*Do you get worried about your exams*?" They were asked to circle one of following to answer the question: Never, Rarely, Sometimes, Frequently.

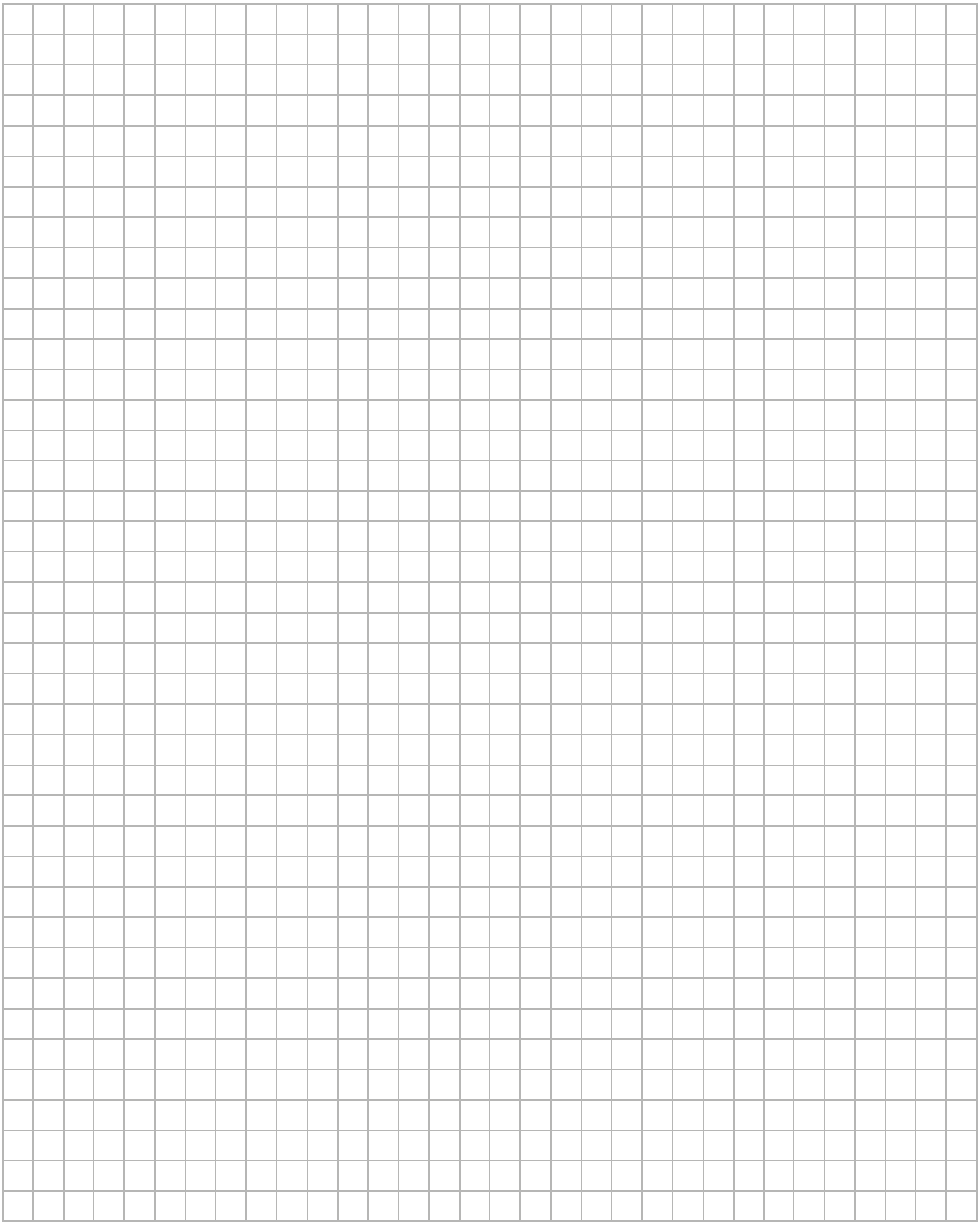
The data below shows the answers from a sample of boys and girls.

Boys
Frequently
Never
Never
Sometimes
Sometimes
Rarely
Sometimes
Sometimes
Frequently
Never
Sometimes
Rarely
Rarely
Frequently
Never
Rarely
Rarely
Frequently
Never
Frequently
Never
Sometimes
Never
Frequently
Rarely
Sometimes
Rarely
Never
Sometimes
Rarely

Girls
Never
Sometimes
Sometimes
Rarely
Never
Frequently
Frequently
Never
Sometimes
Rarely
Frequently
Rarely
Sometimes
Frequently
Frequently
Frequently
Frequently
Rarely
Frequently
Frequently
Frequently
Sometimes
Sometimes
Sometimes
Never
Rarely
Frequently
Frequently
Never
Never
Frequently

(a) How many students were in each sample?

(b) Display the data in a way which allows you to compare the two samples.

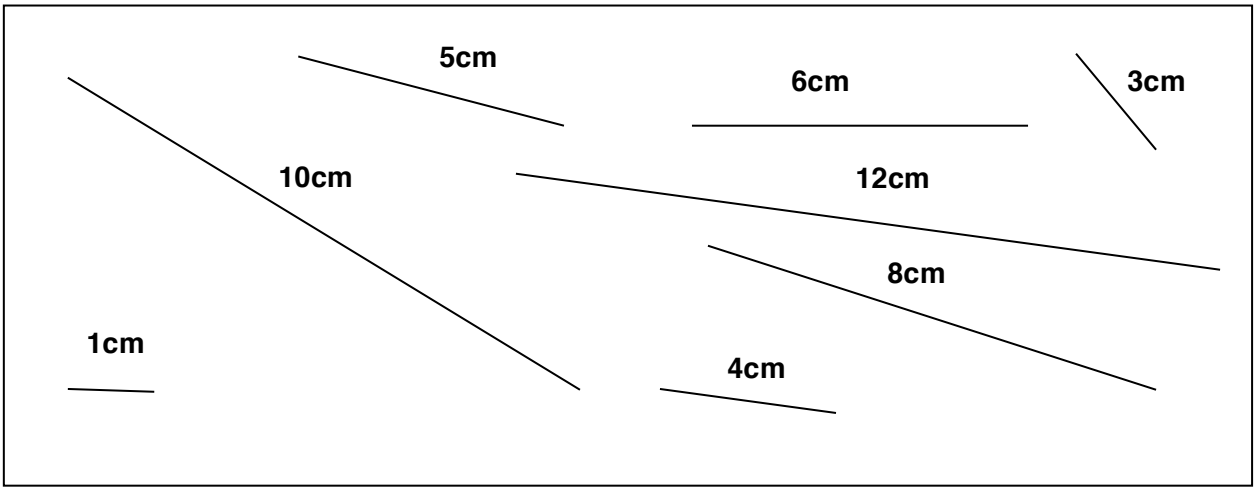


Geometry and Trigonometry

This set of questions; compiled in two documents are intended to help you as you review your work in preparation for the Junior Cycle Mathematics written examination. They are not intended to be exact matches of what will come up in the exam but they should give you a flavour of how the concepts can be examined in context.

Task

Jason had these sticks.



He wanted to make two right-angled triangles.

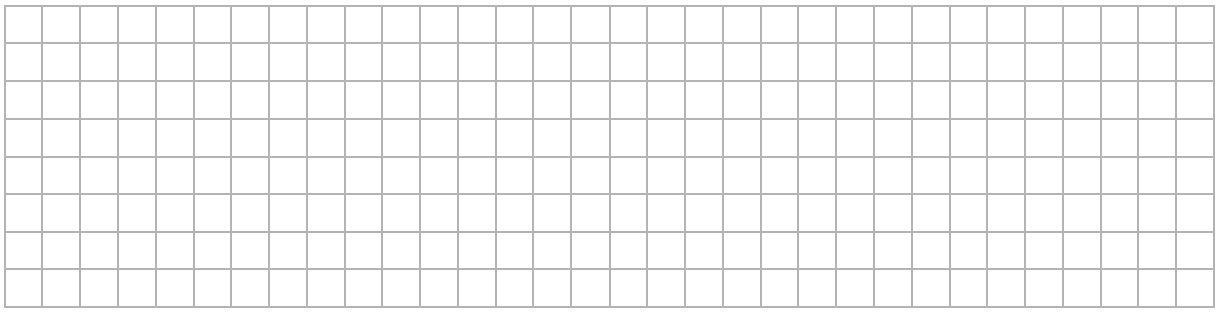
He picked up three sticks and found he could **not** make a right-angled triangle.

(a) Which three sticks might Jason have picked up?

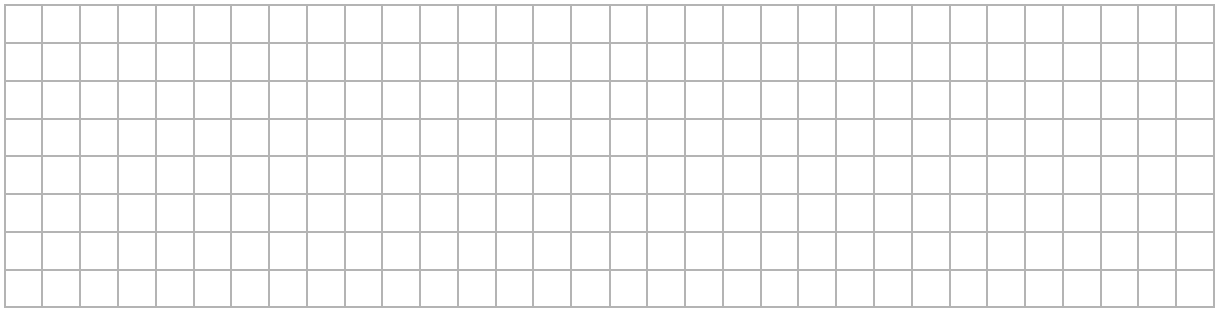
(b) Why did these three sticks not make a right-angled triangle?

Use a theorem from your geometry course to help you explain.

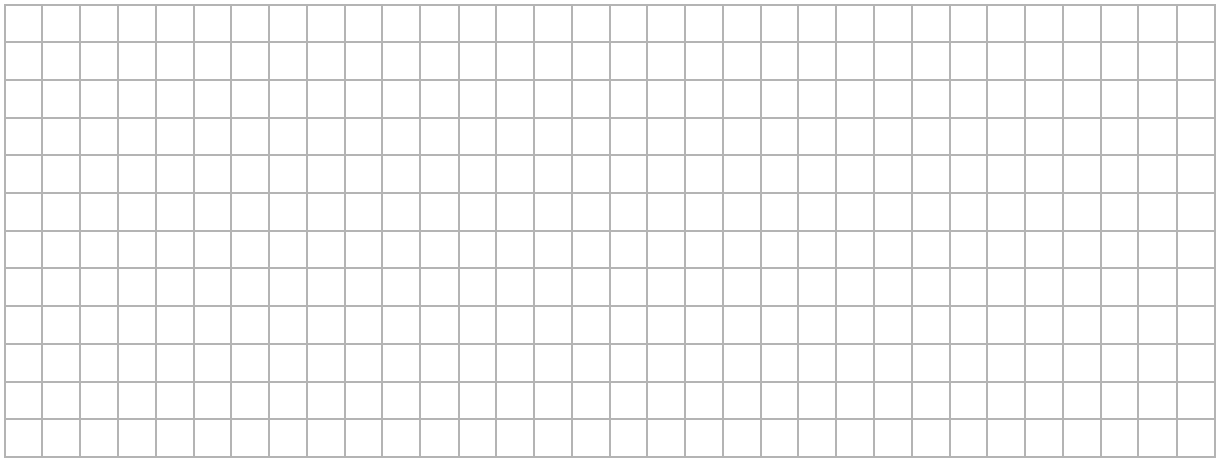
(c) Choose three sticks that will make a right-angled triangle.



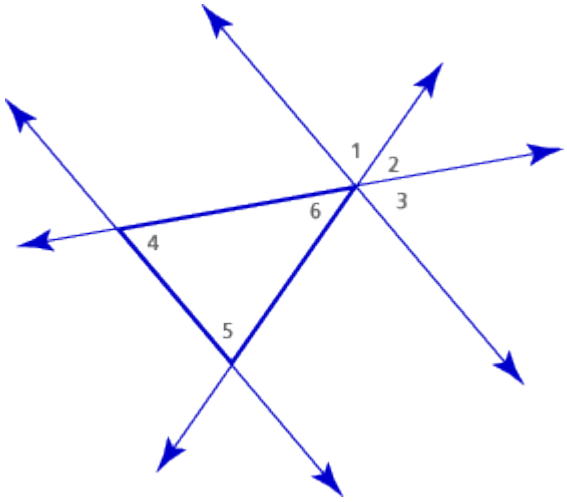
(d) Choose three other sticks which will also make a right-angled triangle.



(e) Show how you know that, in each of these cases, the sticks will make a right-angled triangle.



Task



(a) In the picture above, what is the sum of angles numbered 1, 2 and 3? Explain the reasoning that led to your answer.

(b) In the picture above, the angle numbered 1 is equal in measure to one of the angles in the triangle. Which one?

(c) In the picture above, angle 2 is equal in measure to one of the angles in the triangle. Which one?

(d) In the picture above, the angle numbered 3 is equal in measure to one of the angles of the triangle. Which one?

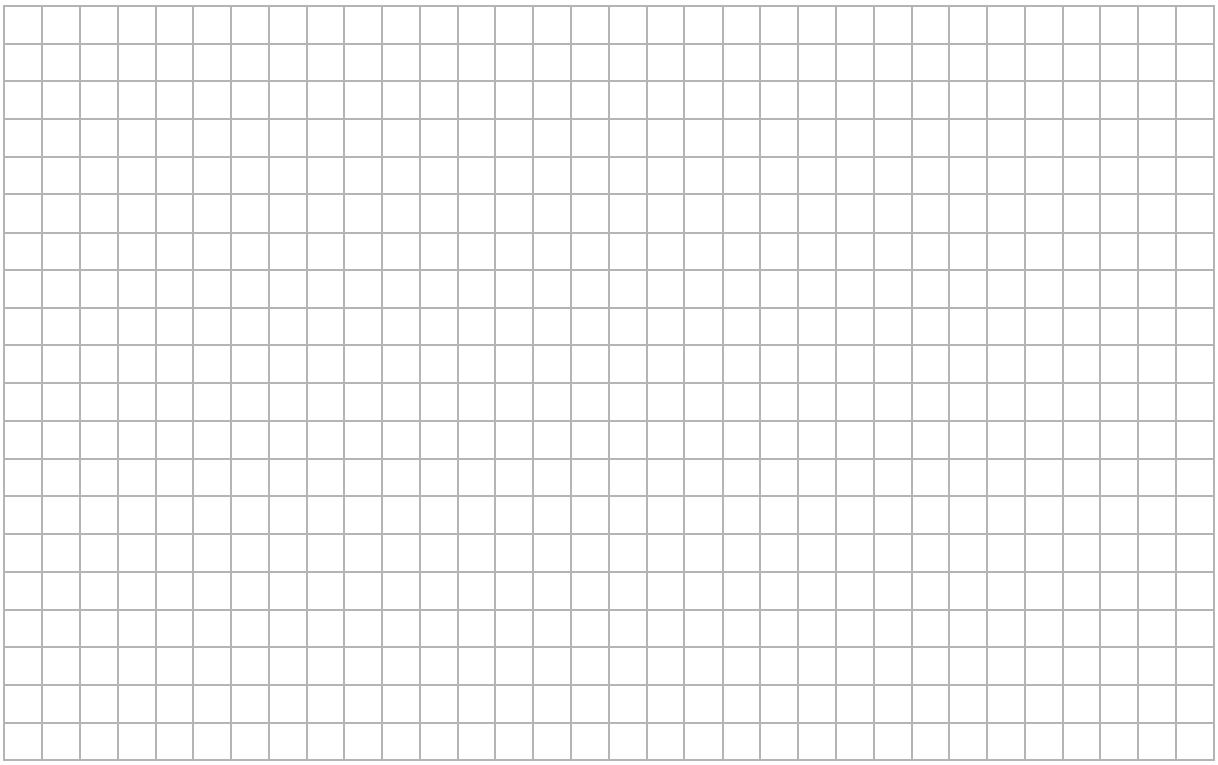
(e) Use your answers to questions (a)-(d) to explain why the sum of the angles numbered 4, 5 and 6 is 180°.

Explain why this would be true for any triangle, and not just the one pictured. Use the following guide.

Given:

To prove:

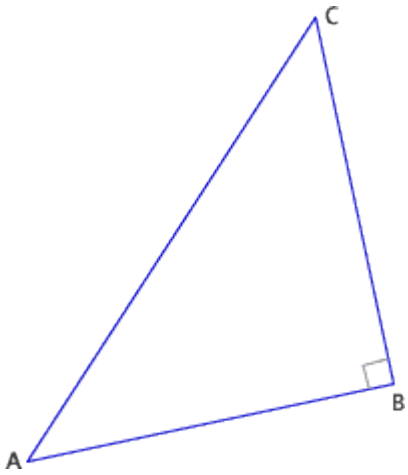
Proof:

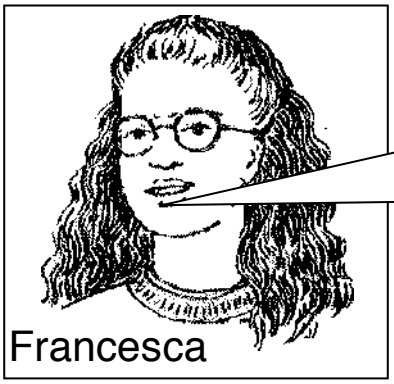


Note: The proof of this theorem is not examinable. However, you should be able to set out your explanation using the sequence of thinking that was involved in the task above.

Francesca and Leo were dissecting shapes and rearranging them to form new shapes. One of their tasks is shown below

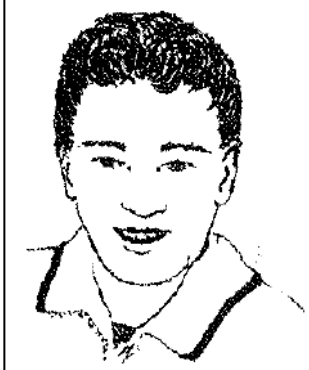
Transform this right-angled triangle into a rectangle by dissecting it and rearranging the parts.





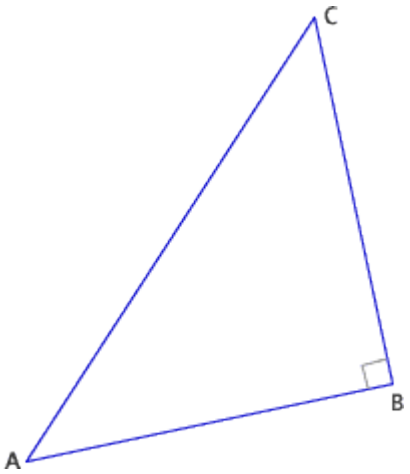
Construct the perpendicular bisector of CB. Label the midpoint of CB as the point D and the point of intersection of the bisector and AC as the point E.

Leo

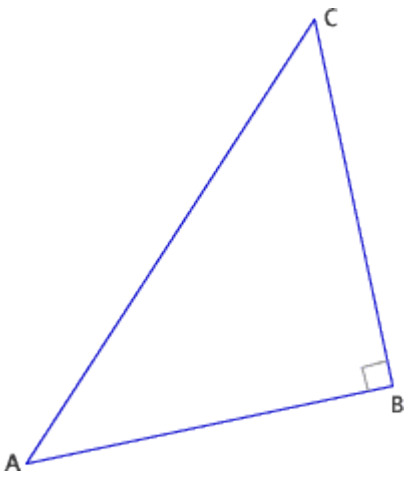


Yes I see…If we cut off the little triangle CED and sort of turn it so that CE lines up with AE. They are equal ‘cos of that **theorem.** Then we will have a rectangle.

On the diagram below accurately follow Francesca's instructions. Show all construction marks clearly.



(a) Accurately complete Leo's instructions in the box below



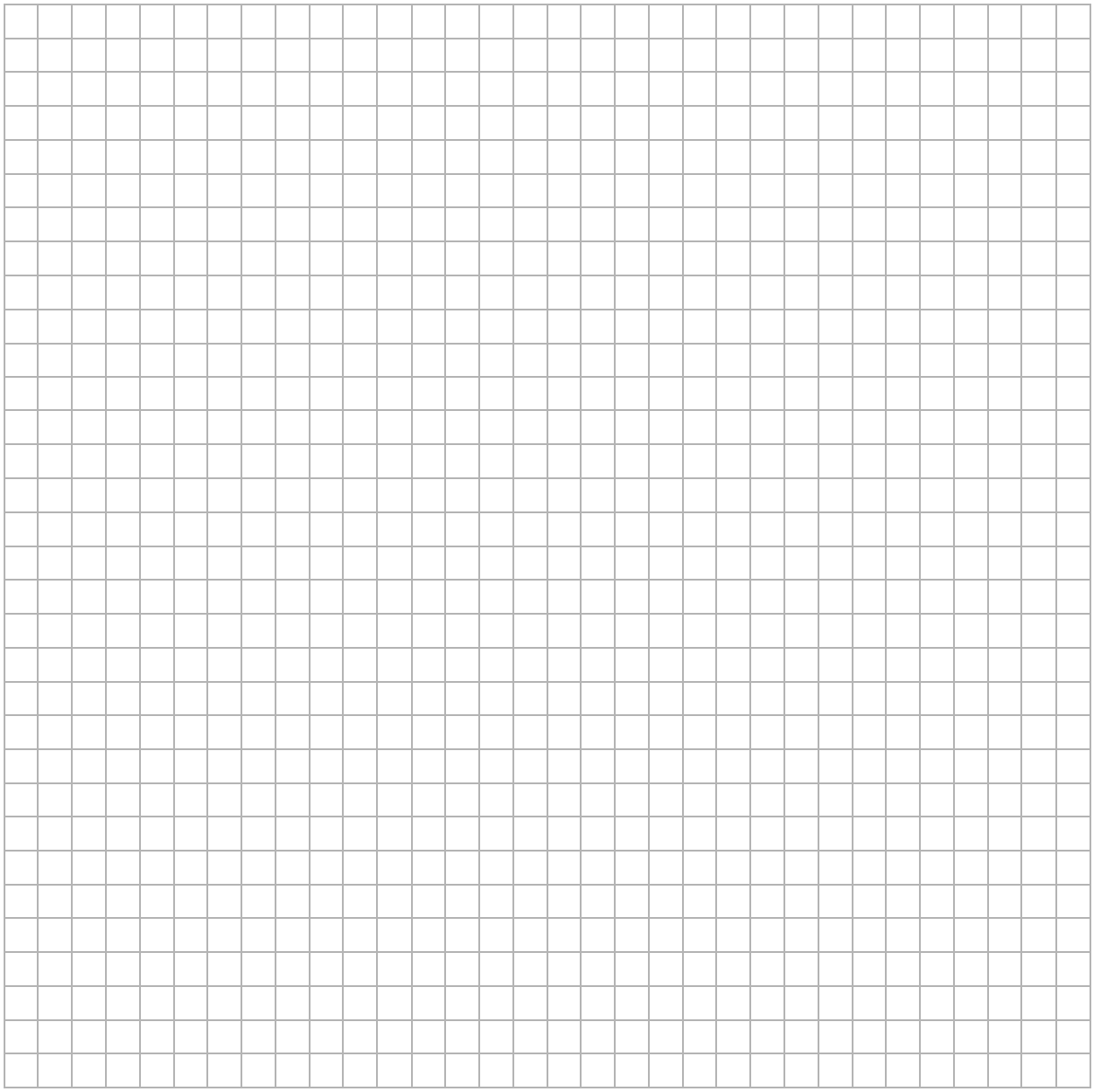
(b) What theorem was Leo referring to when he said " *..CE will line up with AE. They are equal 'cos of that theorem…"*

Why are CE and AE equal?

(c) Leo says that the re-arranged shapes will make a rectangle.

Do you agree with Leo?

Explain your thinking. You will need to write down some **properties** of a rectangle and show how the figure Leo ends up with has these **properties**.



Q Calculate the height of

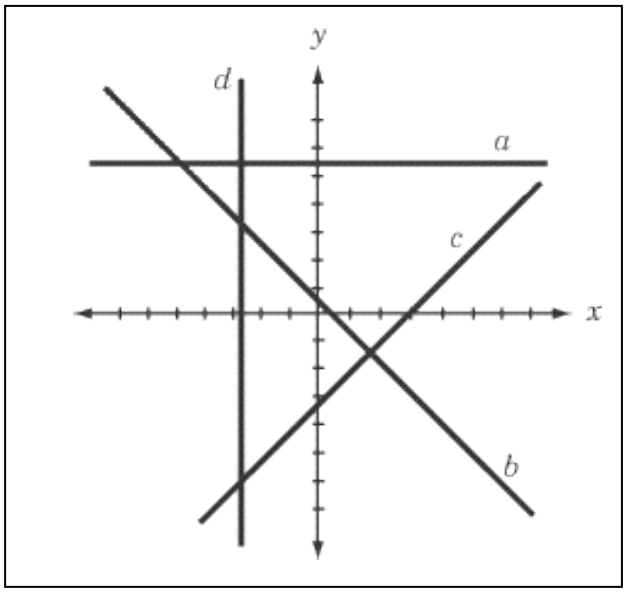
(a) an equilateral triangle of side length x

(b) an isosceles triangle of side lengths x and y

What are the properties of an isosceles triangle? Sketch the triangle and mark the length of each of the sides.

Task

Of the four lines pictured below, one has a slope of 0, one has a slope of 1, another has a slope of –1, and another has an undefined slope. Complete the table to show which is which.

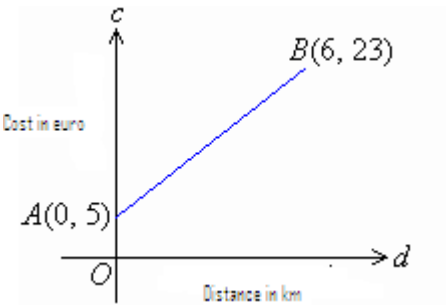


Line *a* has slope:	
Line *b* has slope:	
Line *c* has slope:	
Line *d* has slope:	

Give reasons for your choices.

Task

The cost of transporting documents by courier can be represented by the following straight line graph.



Use the graph to help you work out how the courier charges customers.

How much would the courier charge to transport documents a distance of 30km?

Eoin's company is working to a tight budget. They need to transport documents 15km across the city.

Eoin gets a quote from another company who claim to be cheaper. Their advertisement reads

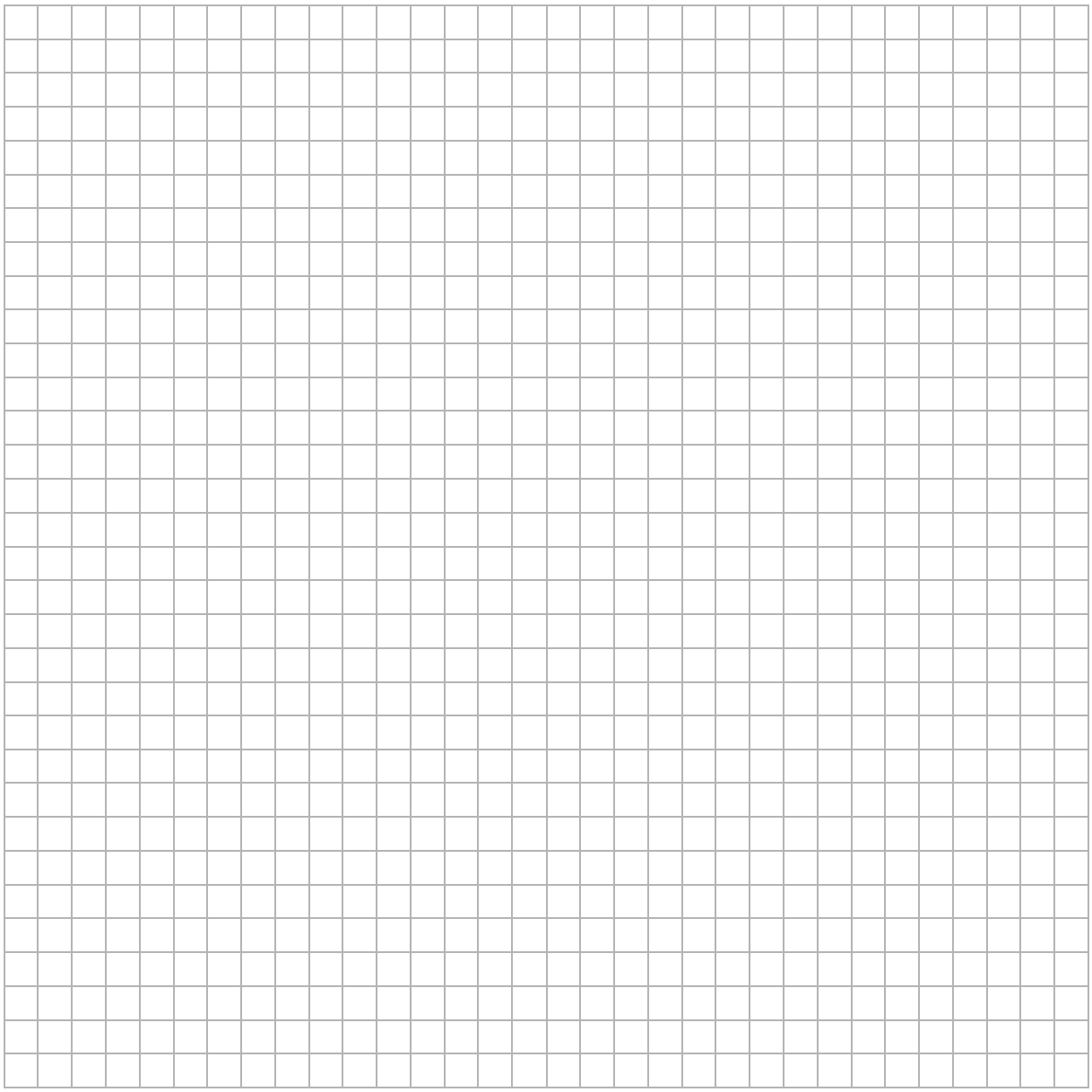
Cheapest courier in town

We charge €3.50/ km and no standing charge.

Which courier should Eoin ask to transport his documents across the city? Justify your decision by comparing the prices charged for this job by both companies.

Can the second company stand by their claim of being the cheapest courier in town?

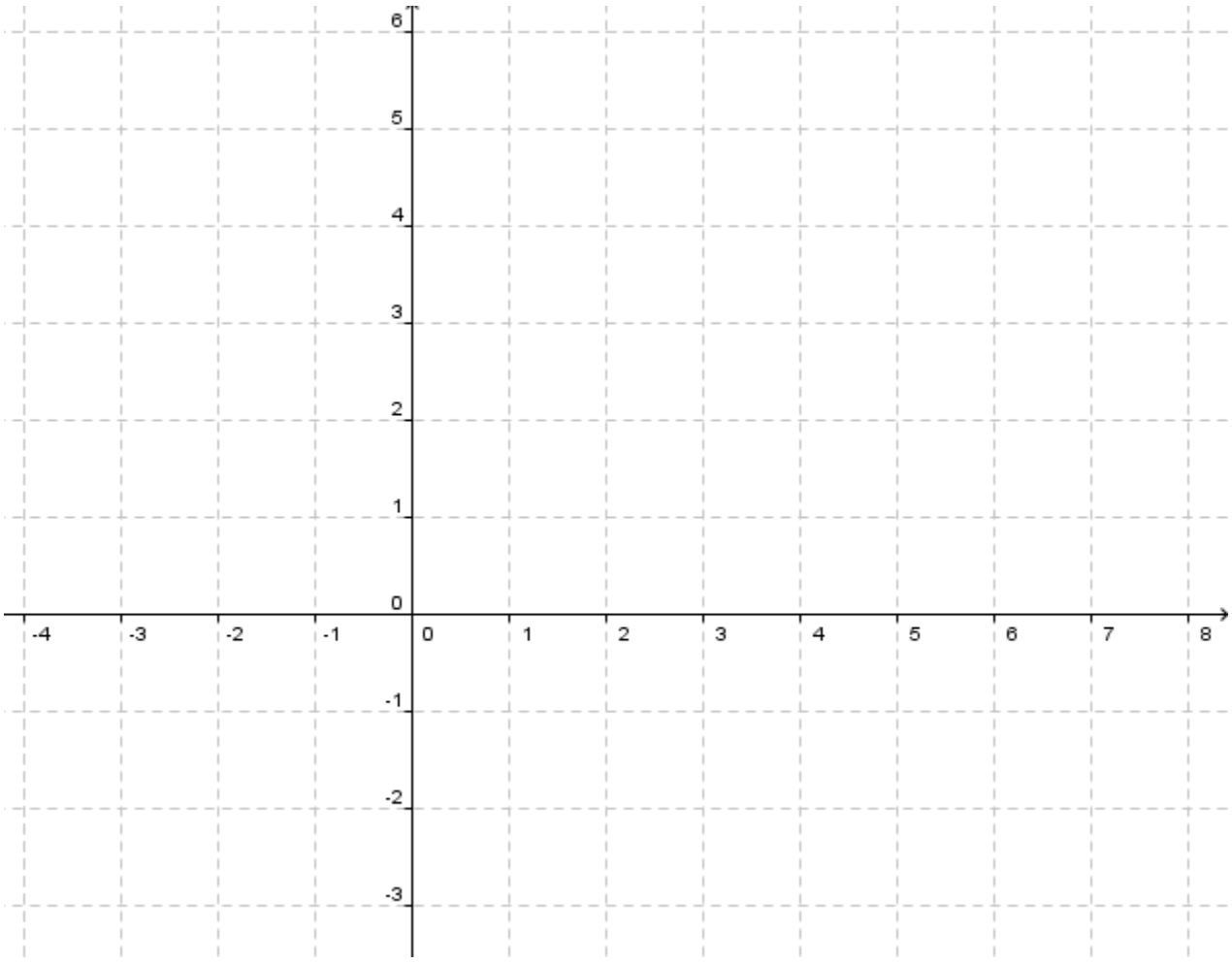
Justify your answer by referring to a graphical representation of each company's charges.



Task

Draw the following shapes on the coordinate axes.

- a square
- a right angled triangle
- an isosceles triangle
- a parallelogram



Write down the co-ordinates of the **vertices** of each shape

Square (….., ….) (….., ….) (….., ….) (….., ….)

Right- angled triangle (….., ….) (….., ….) (….., ….)

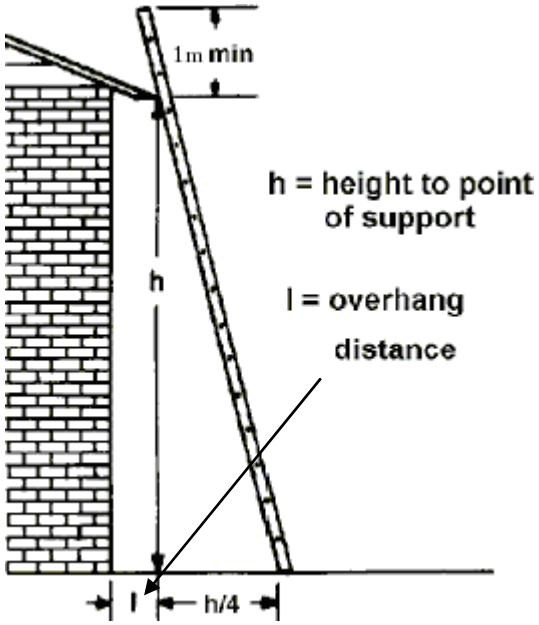
Isosceles triangle (….., ….) (….., ….) (….., ….)

Parallelogram (….., ….) (….., ….) (….., ….) (….., ….)

Task

You're locked out of your house and the only open window is on the second floor, **7m** above the ground. You need to borrow a ladder from one of your neighbours. There's a bush along the edge of the house, so you'll have to place the bottom of the ladder **3m** from the house. What length of ladder do you need to reach the window?

Sketch a mathematical diagram. Use straight lines to represent the **wall of the house**, the **ladder** and the **ground**. Mark each line with the correct measurement. If you do not know the measurement mark it x. Use your geometry to calculate the length of the ladder needed.

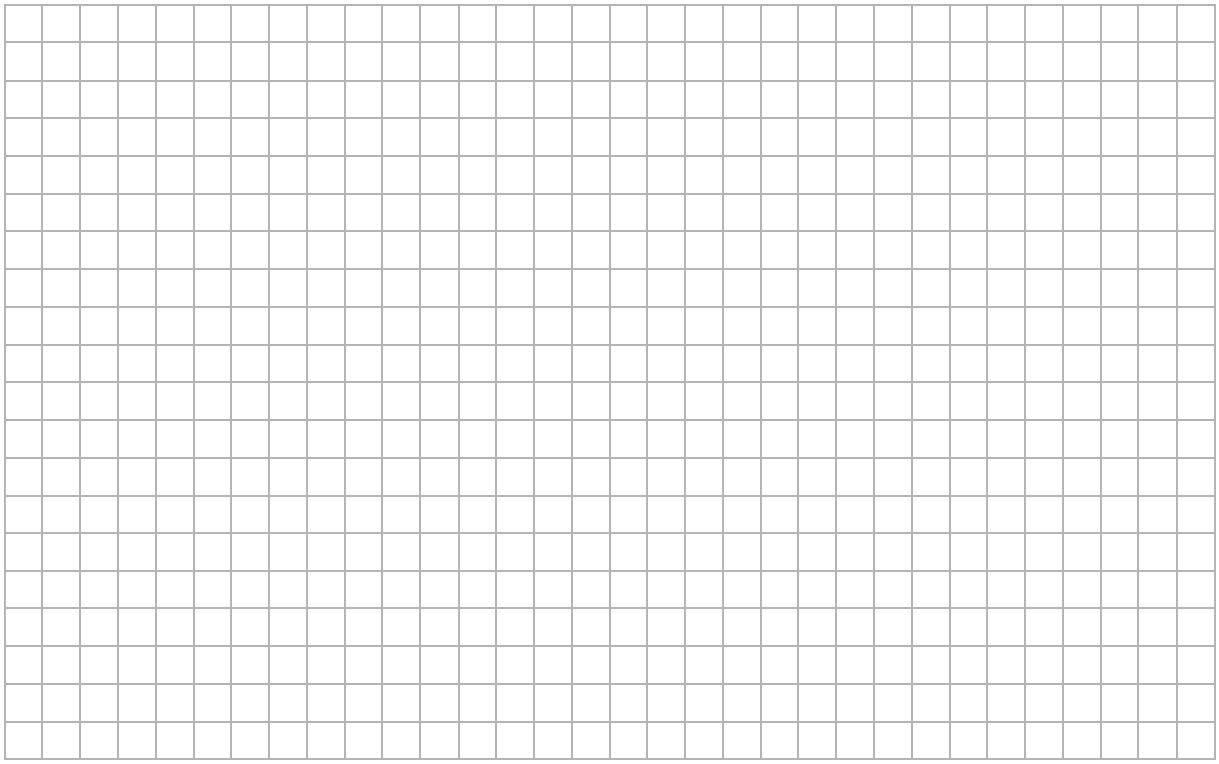


The diagram is taken from a safety manual that accompanies a particular brand of ladder.

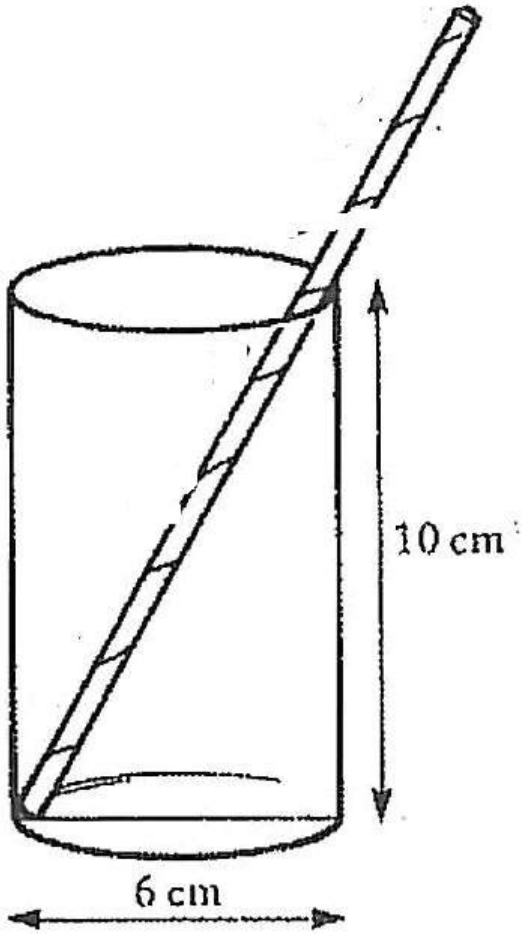
You can see from the diagram that the length of a ladder that you would choose for a particular job depends on the height of the object it will be leaning up against.

Generalise this relationship.

Does the angle the ladder makes with the horizontal depend on the height of the object it is leaning up against? Explain your answer.



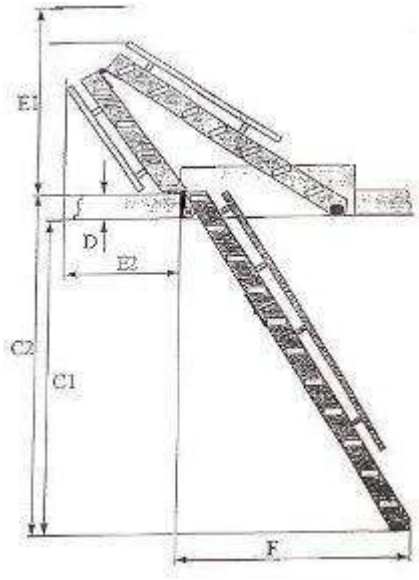
Task



The straw is 20cm long.
Calculate the length of the straw sticking out from the top of the glass.

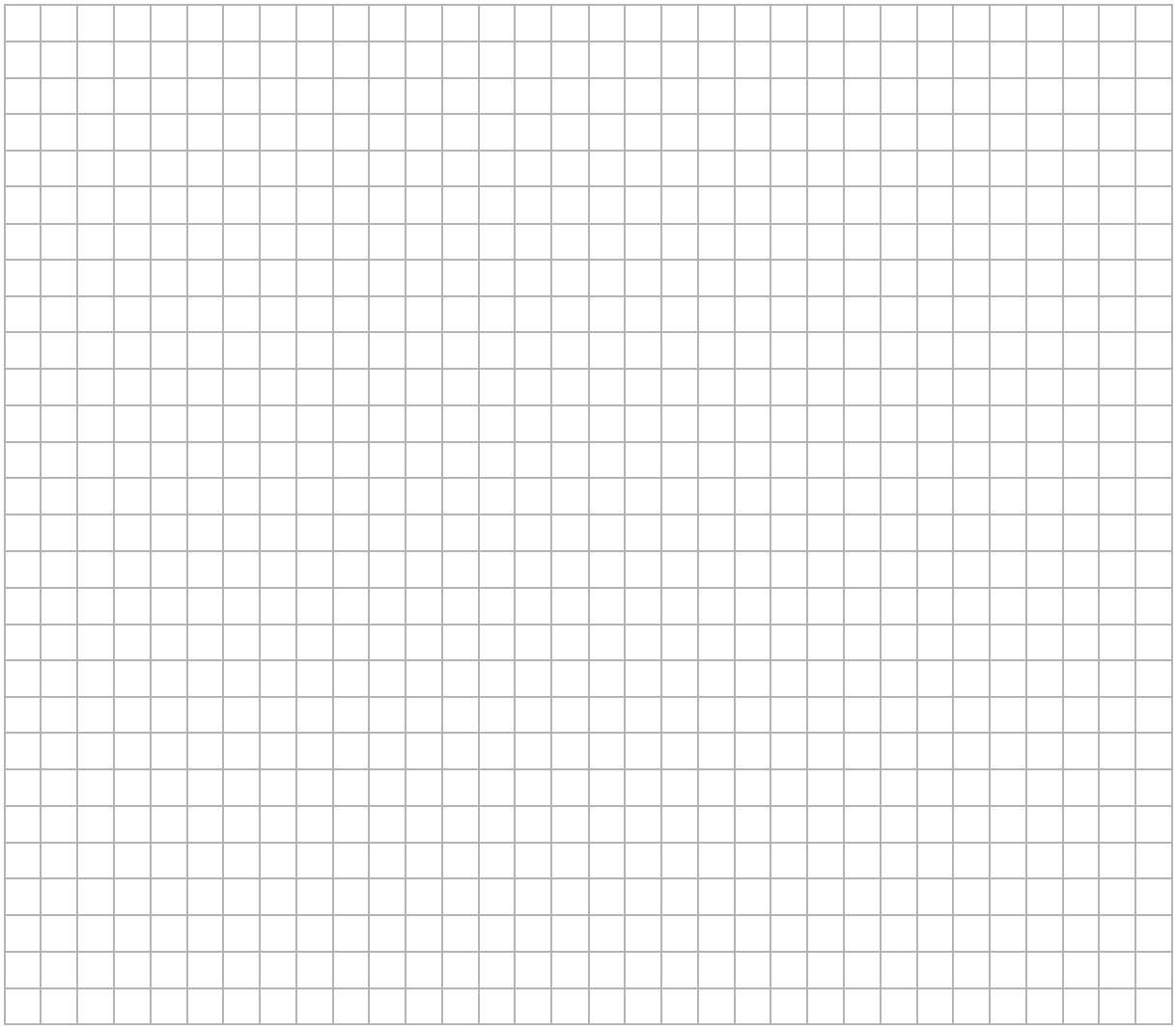
Task

An installation guide for the *Sandringham Electric Attic ladder* is shown below



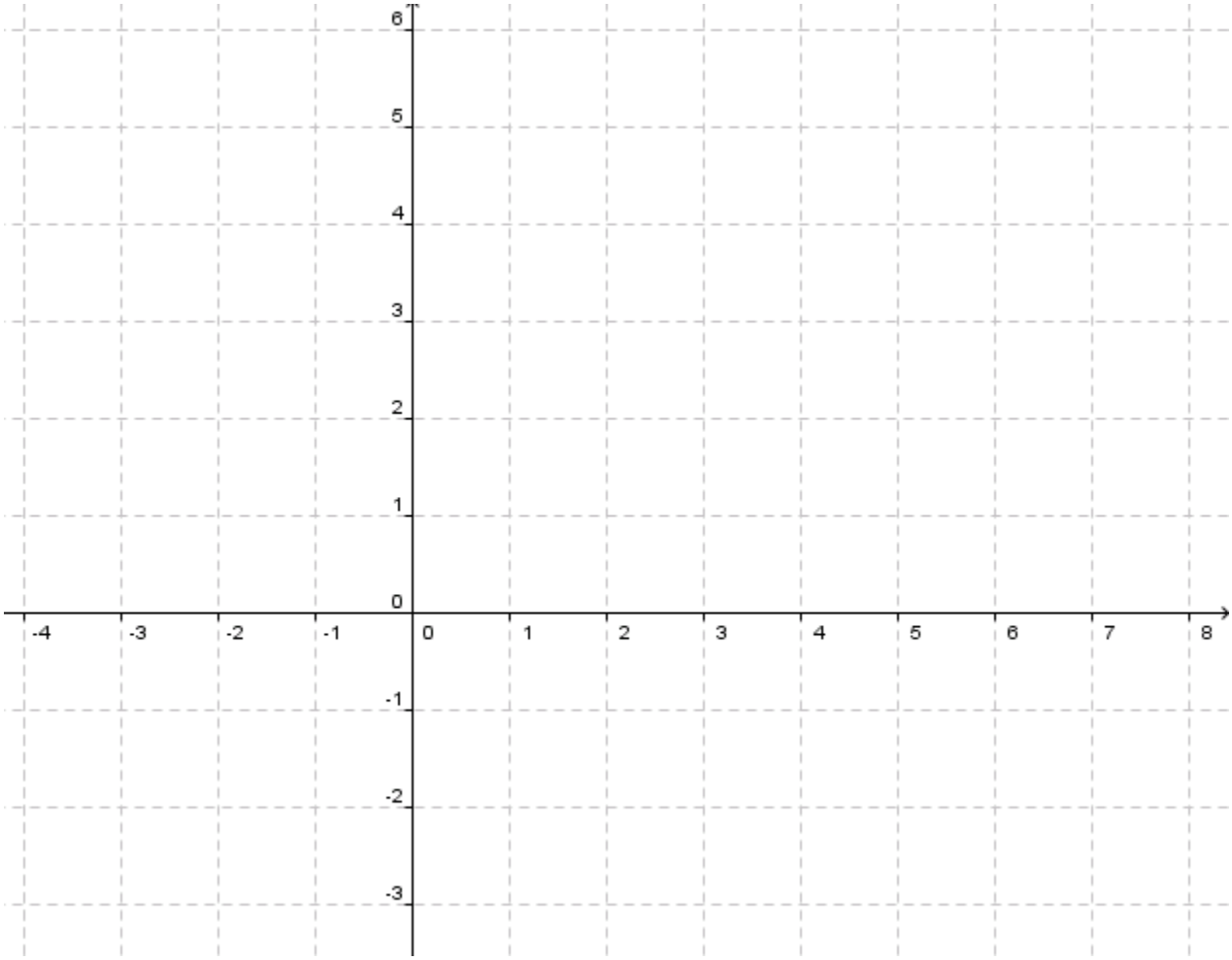
Ladder Size	Floor to Floor Height C_2	Storage swing and Height	Horizontal Distance F
	250cm	145cm	159cm
Length [ink] cm	260cm	155cm	166cm
[ink]	270cm	[ink] cm	173cm
	280cm	175cm	180cm
Length 150 cm	[ink] cm	146cm	166cm
[ink]	280cm	166cm	[ink] cm

Some ink has spilled on the table. Use your mathematics to find the lengths covered by the ink blots. If you are unable to calculate a particular missing length, explain why you are unable to do so.



Task

Plot the points (-1, 2), (5,2), (-1,-1) (5,-1) on the grid.



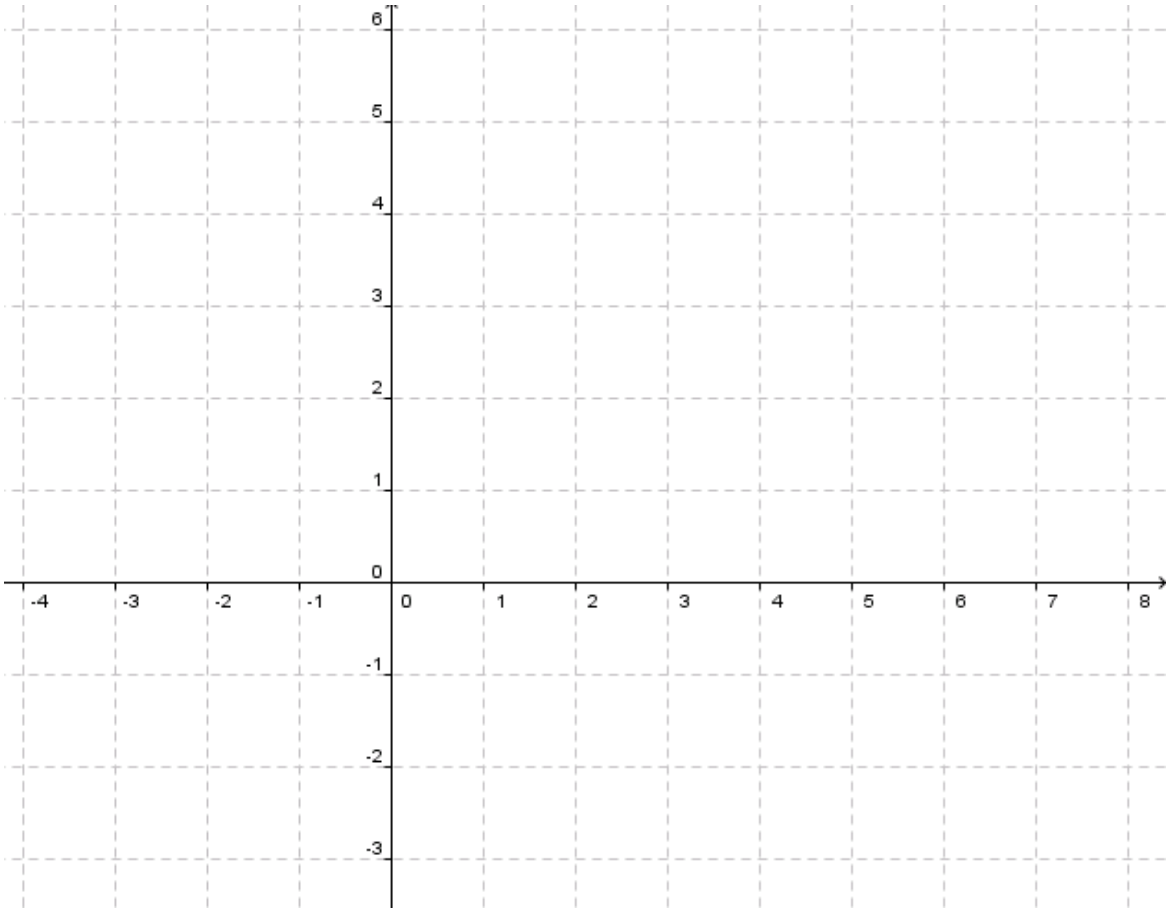
Join them to form a shape.

What is the name given to this shape?

Write down two properties of that shape.

Task

Plot the points (-1, 2), (5, 2), (-1, -1) (5, -1) on the grid.

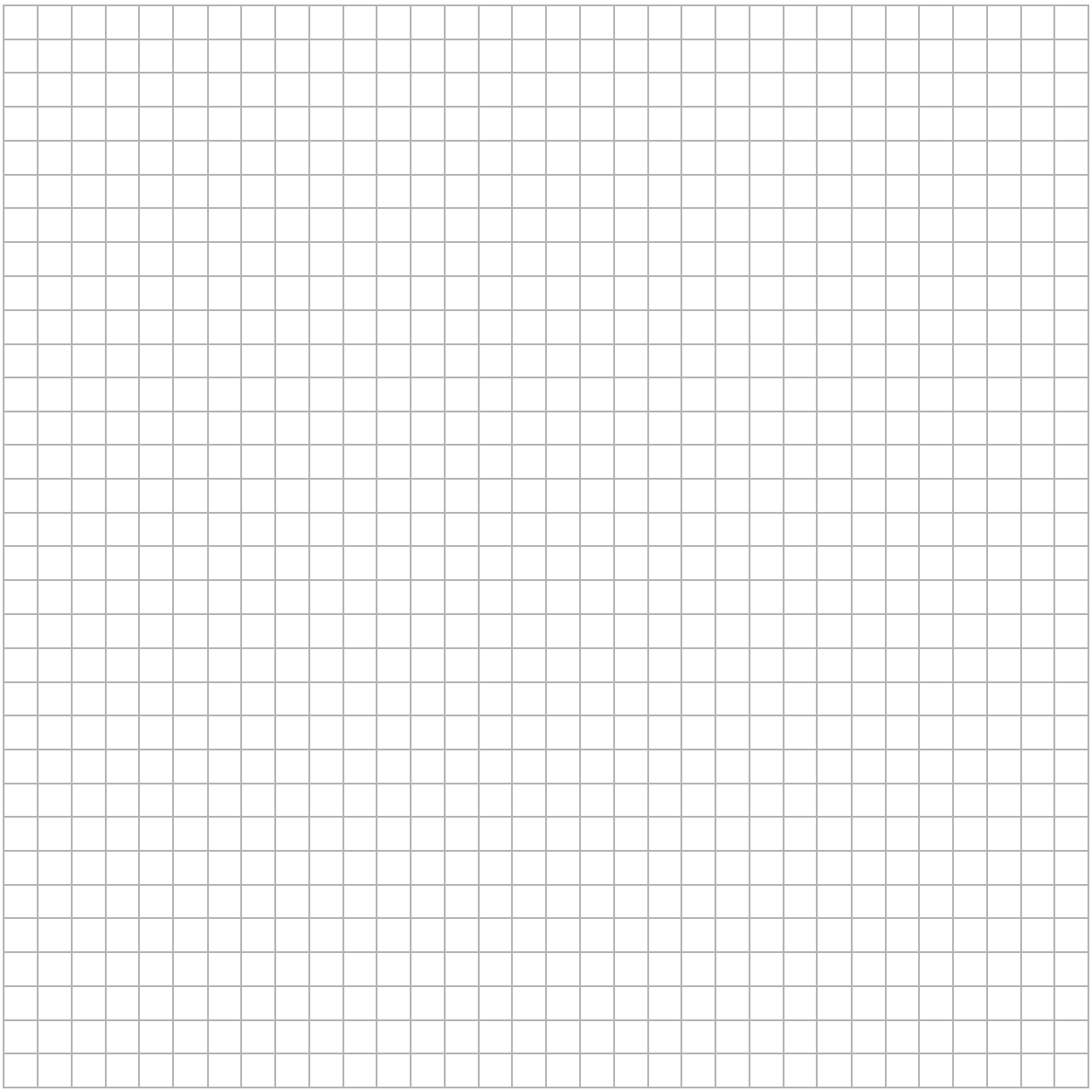


Join them to form a shape.

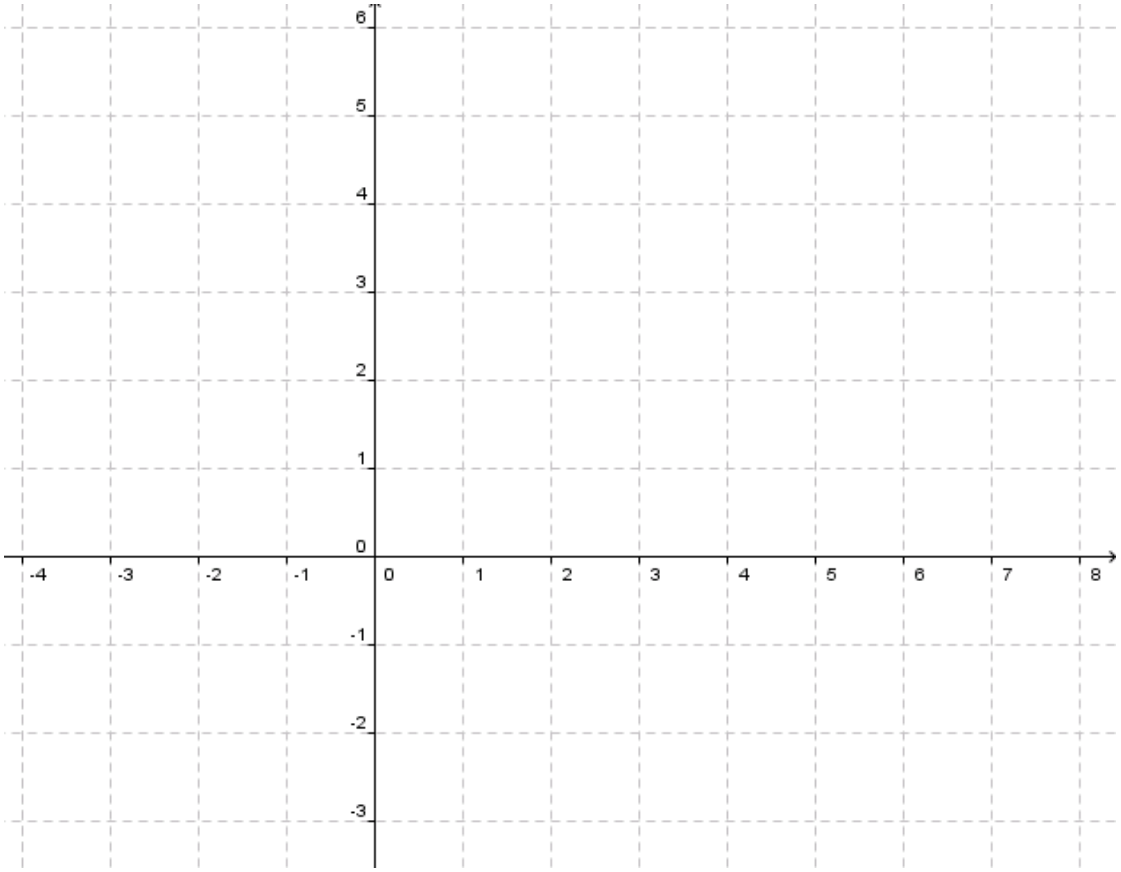
What is the name given to this shape?

Write down two properties of that shape.

Prove that the shape you have made on the grid has those properties.



Plot the points (8, 2), (8,-4), (2,-1) on the grid.

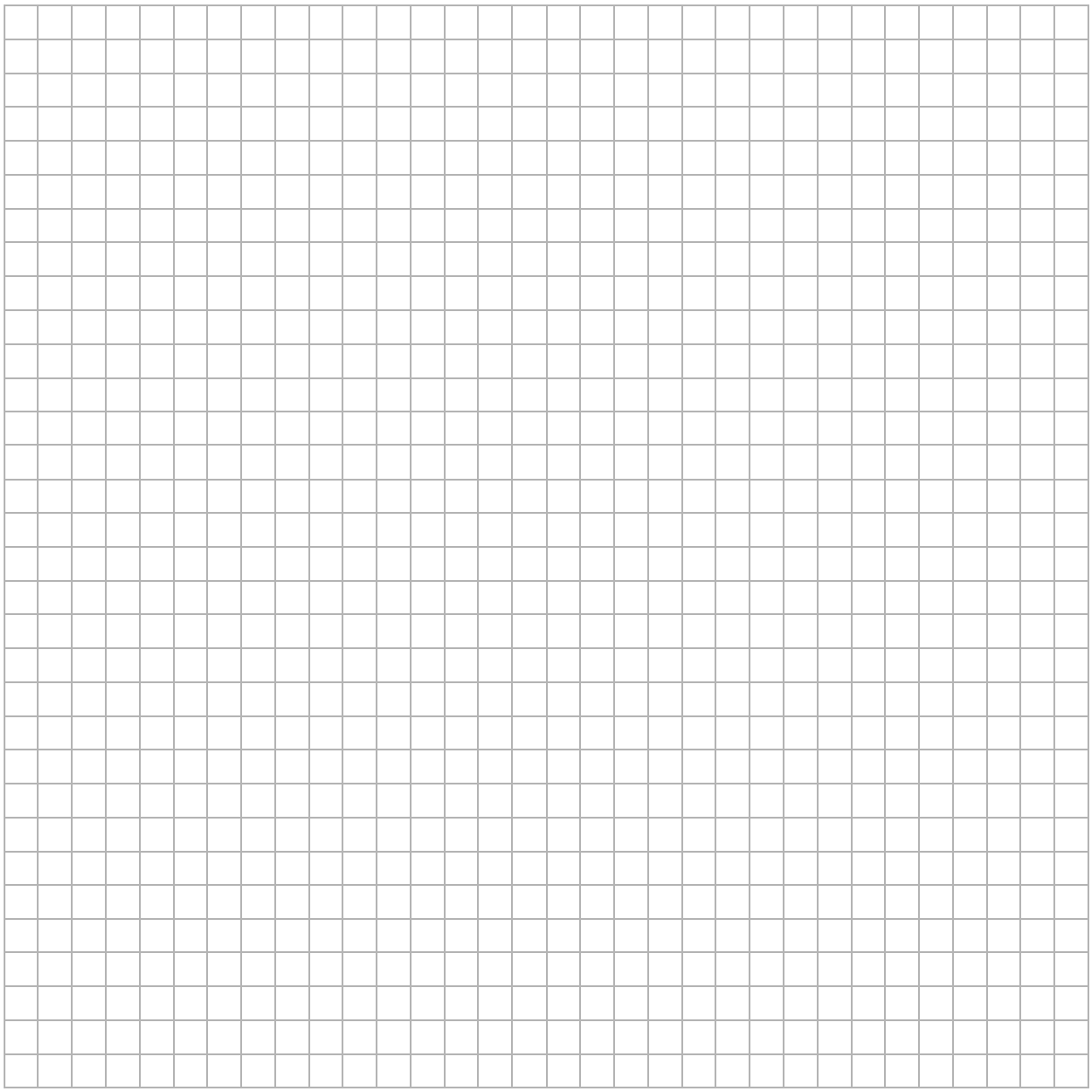


Join them to form a shape.

What is the name given to this shape?

Write down two properties of that shape.

Prove that the shape you have made on the grid has those properties



Task

Say which of the following is true by ticking the correct box

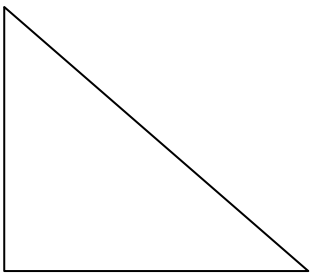
In the diagram below:

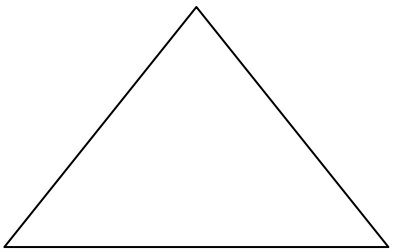
One **F** is the image of the other after an axial symmetry ☐

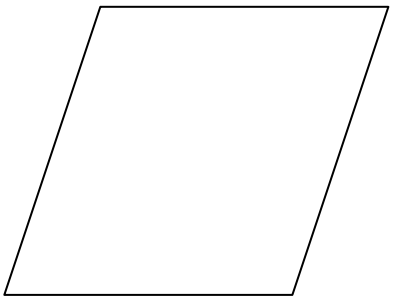
One **F** is the image of the other after a central symmetry ☐

One **F** is the image of the other after a translation ☐

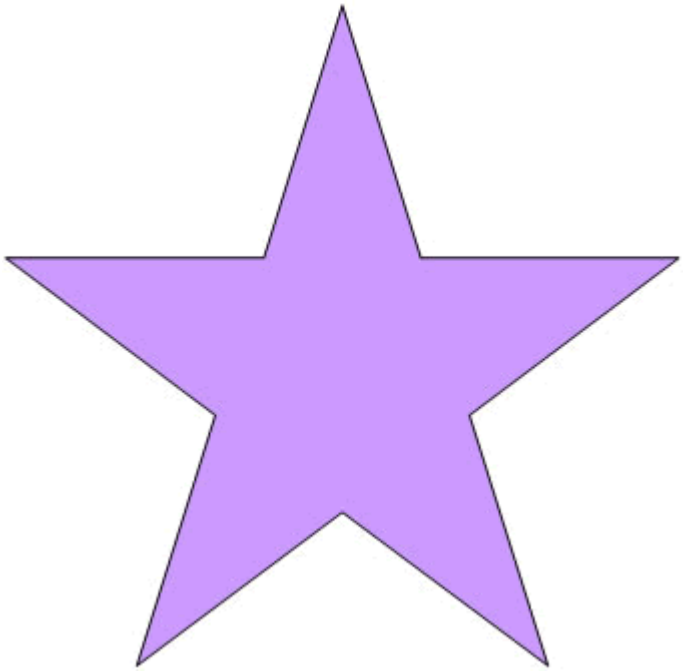
Draw as many lines as symmetry as possible for each figure below.







Use tracing paper or fold the shape to help find the lines of symmetry. Look for patterns or properties of these lines.

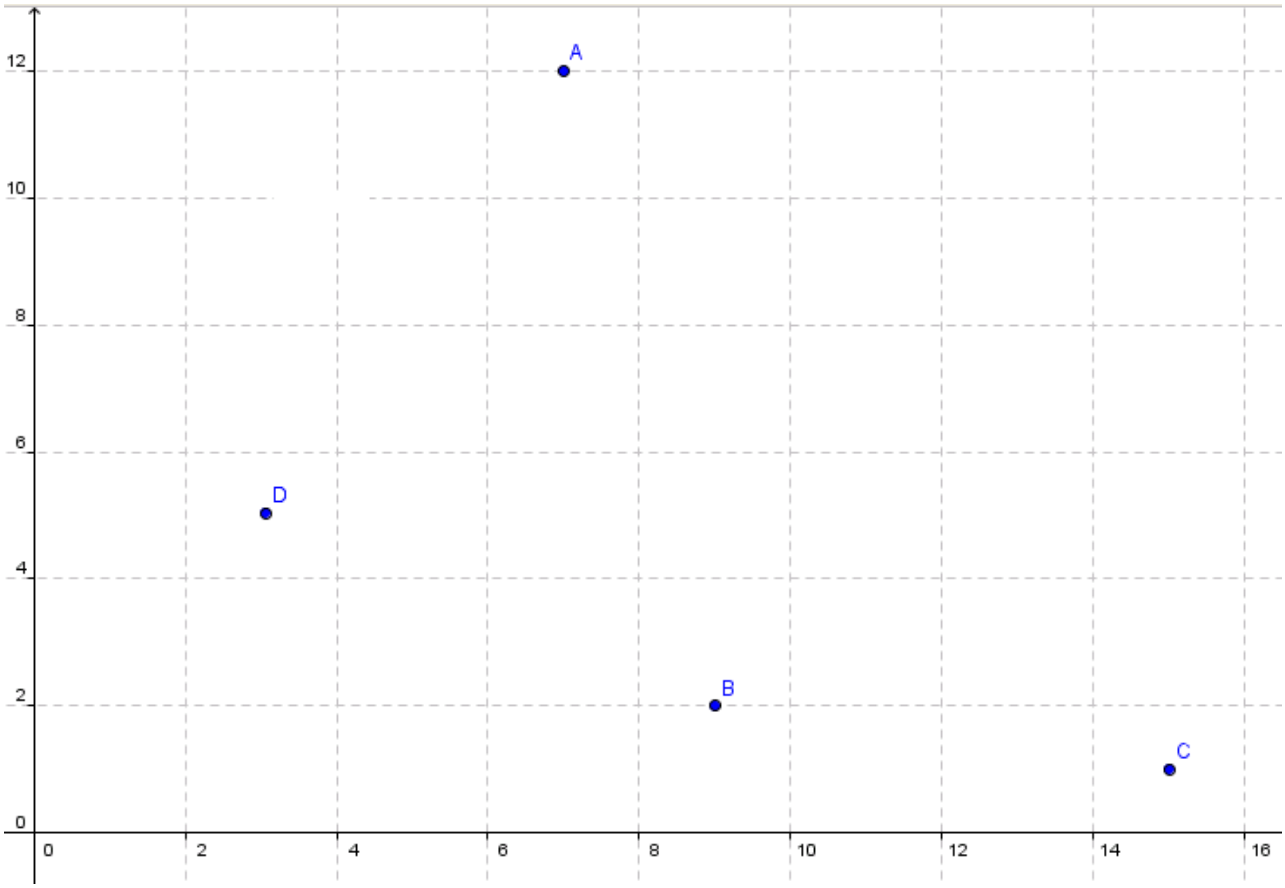


Task

A monorail similar to the one shown was planned for an amusement park.

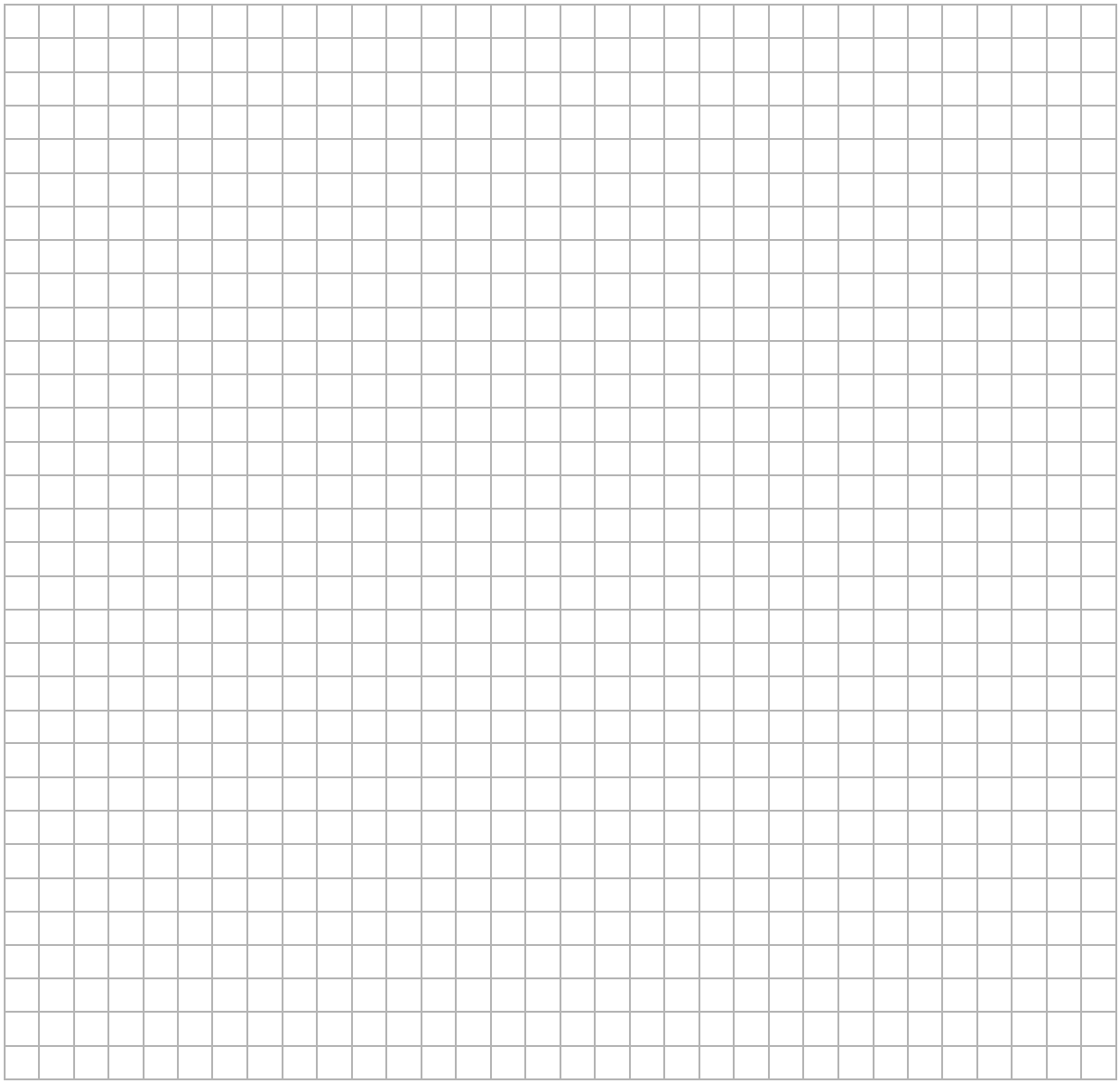
The original plans had the supports located as shown on the grid below.

A (7, 12) B (9, 2) C (15, 1) D (3, 5)



In order to make room for a car park, engineers have decided to demolish the supporting pillar C and relocate it.
They have also decided that, on the plans, the new support pillars should be able to form a parallelogram

Plot the new location of the supporting pillar and write its coordinates. Label it $\mathbf{C_1}$.
Use the definition or properties of a parallelogram to verify that the new layout is a parallelogram. You must use the slopes of the sides, the lengths of the lines or both to verify your answer.



Task

Linda Armstrong is a professional forester.

In order to calculate how much wood is in a forest she must measure the height of the trees.

We don't count and measure every tree in the forest. That would simply take too long. Foresters measure a few bits of the forest and, on the basis of those bits, estimate what the whole forest contains

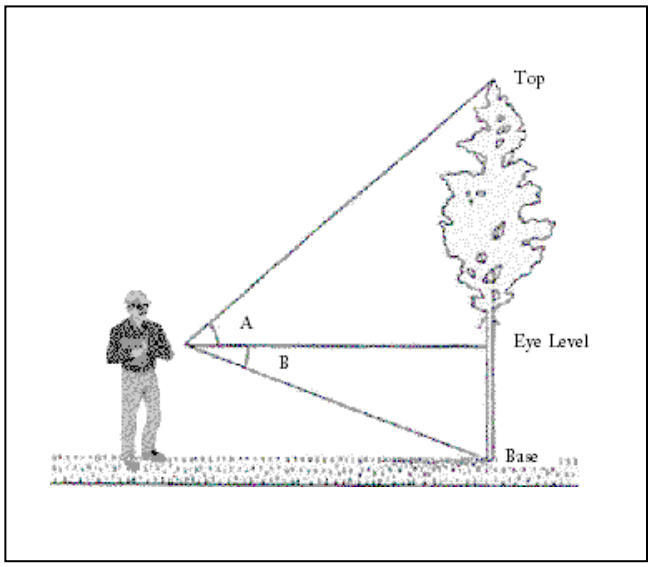
Suggest a way that Linda could choose which "bits" to **sample.**

Foresters need to monitor the growth of trees. They measure their heights each year and can determine the **yearly tree growth.**

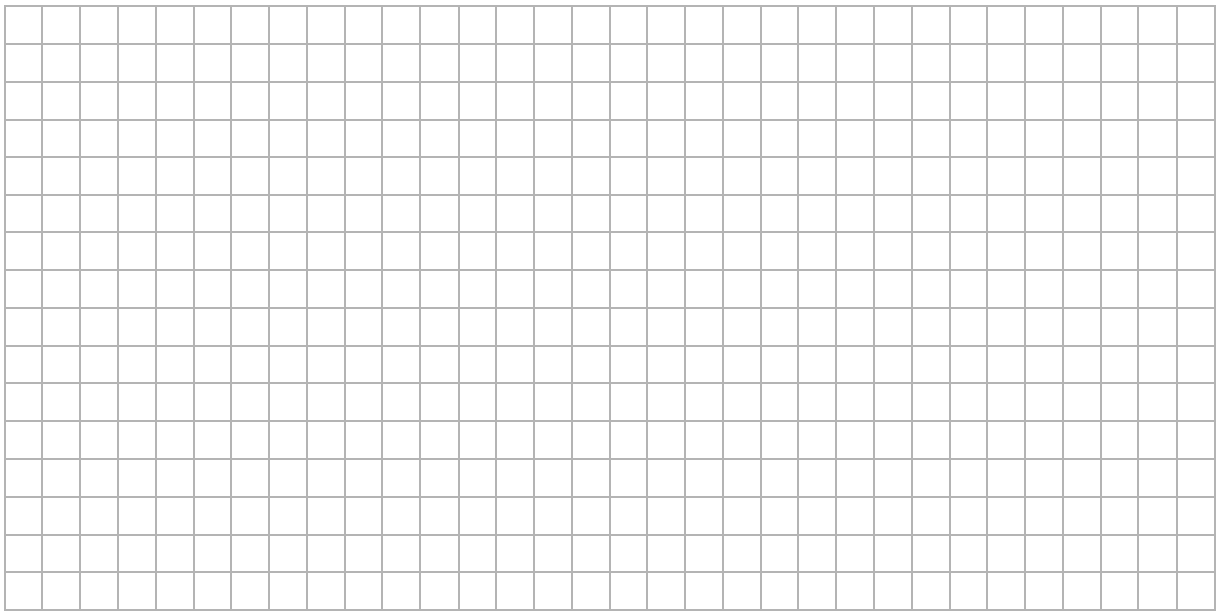
You can determine the tree's height by using trigonometry. If you measure the horizontal distance between yourself and the tree, and measure the angles leading to the tree's top and its base, using a simple instrument called a clinometer, you have enough information to calculate the tree's height.

Linda used this technique and obtained the measurements in the table

Angle A	55°
Angle B	25°
Distance from Linda to Tree	2.5m



Use trigonometry to calculate the height of the tree.



Linda wanted to compare the growth of trees on a tree farm with the growth of trees in a forest. The stem and leaf plot shows the yearly growth, in cm, of a selection of trees in both the tree farm and the forest.

Tree Farm		Forest
1	1	0 1 3
3 3	2	1 5 7
7 2 1	3	0 1 3 8 9 9
9 8 0	4	2 3 4 4 8
1 0	5	0 1 3 7

| 2 | 5 = 25cm

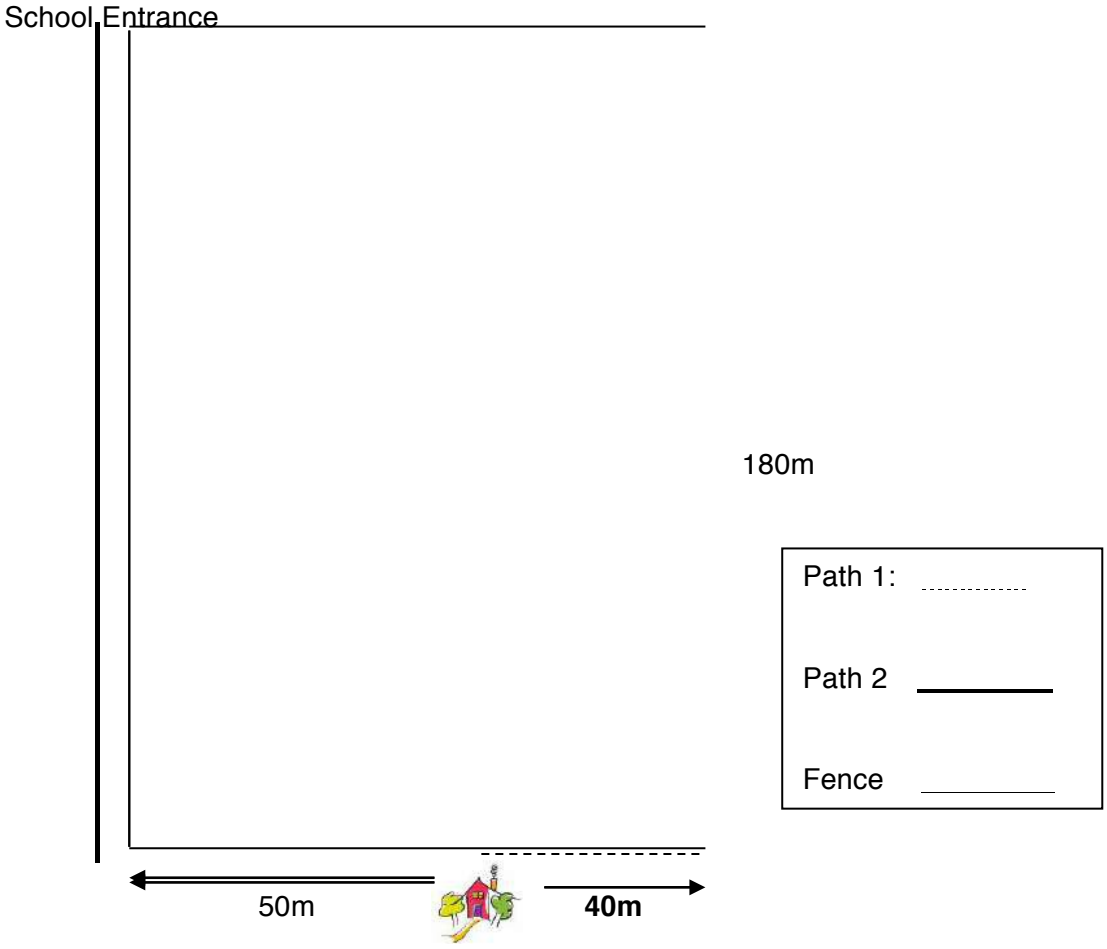
1 | 5 | = 51cm

What is the difference between the **median** yearly growth in cm of the selection of trees from the forest and those from the tree farm?

Is there any evidence to suggest that the trees on the farm grow quicker than the trees in the forest?

Task

Mark's house is located near the **perimeter fence** of his school playing field.



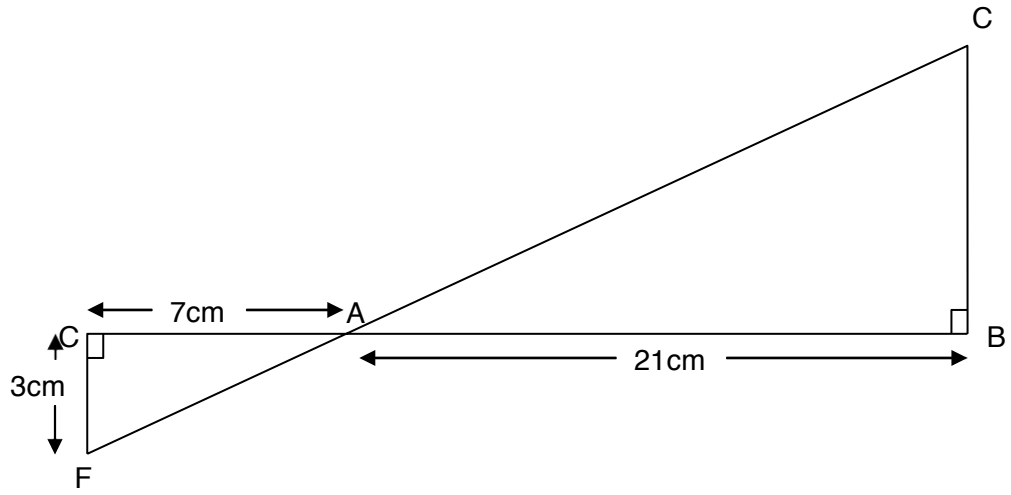
There are two paths Mark can take to school. He can walk along the fence, go through the gate to the playing field and walk across the field (Path 1), or walk around the perimeter fence (Path 2).

What is the difference in distance between the two paths?

Task

In the diagram below, line segments CF and BE intersect at

A. Is AEF similar to ACB?

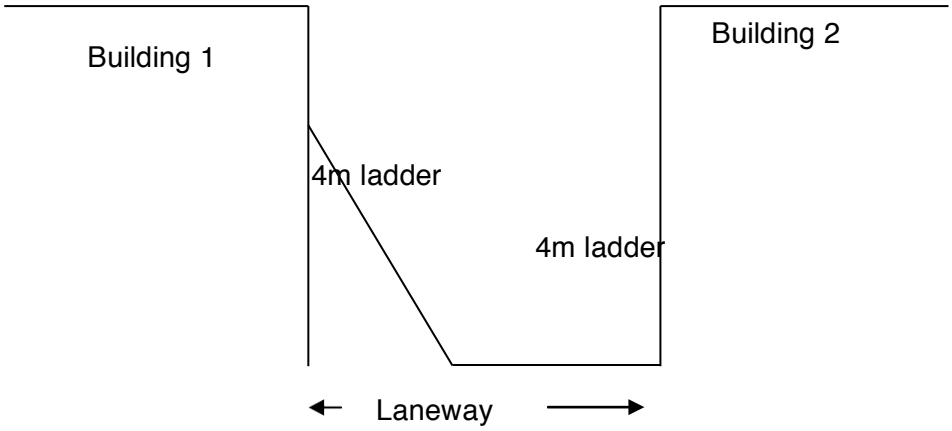


Give a reason for your answer.

Calculate the lengths AF, AC and CB.

Task

Jack placed a 4m ladder in a laneway between two buildings



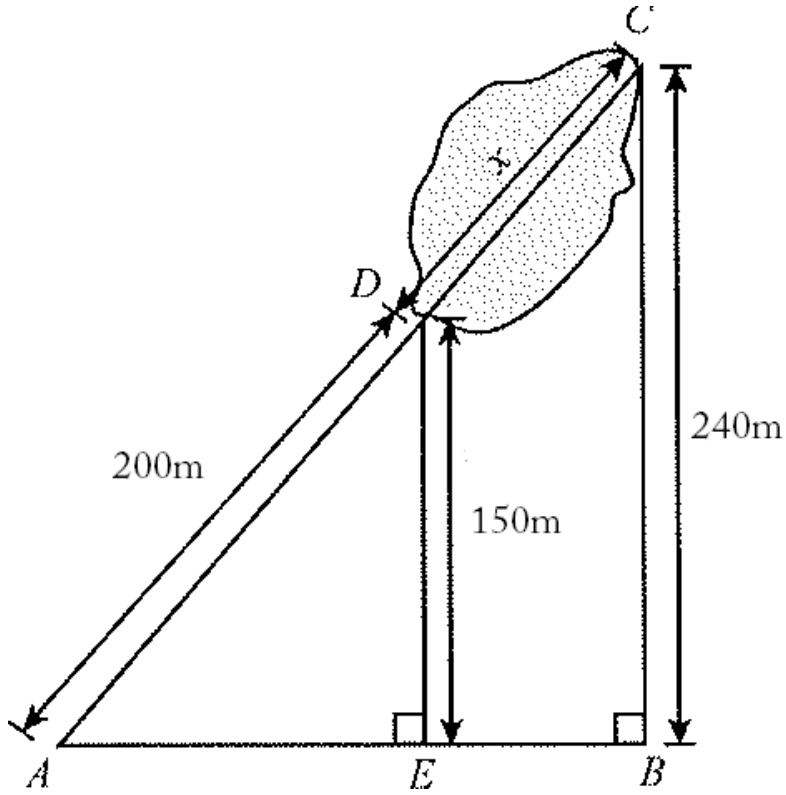
When he tilted the ladder one way it reached 2.5m up the wall of building 1 and when tilted the other way it reached 1.5m up the wall of building 2.

What is the width of the laneway?

Task

A surveyor wants to determine the distance across a lake.

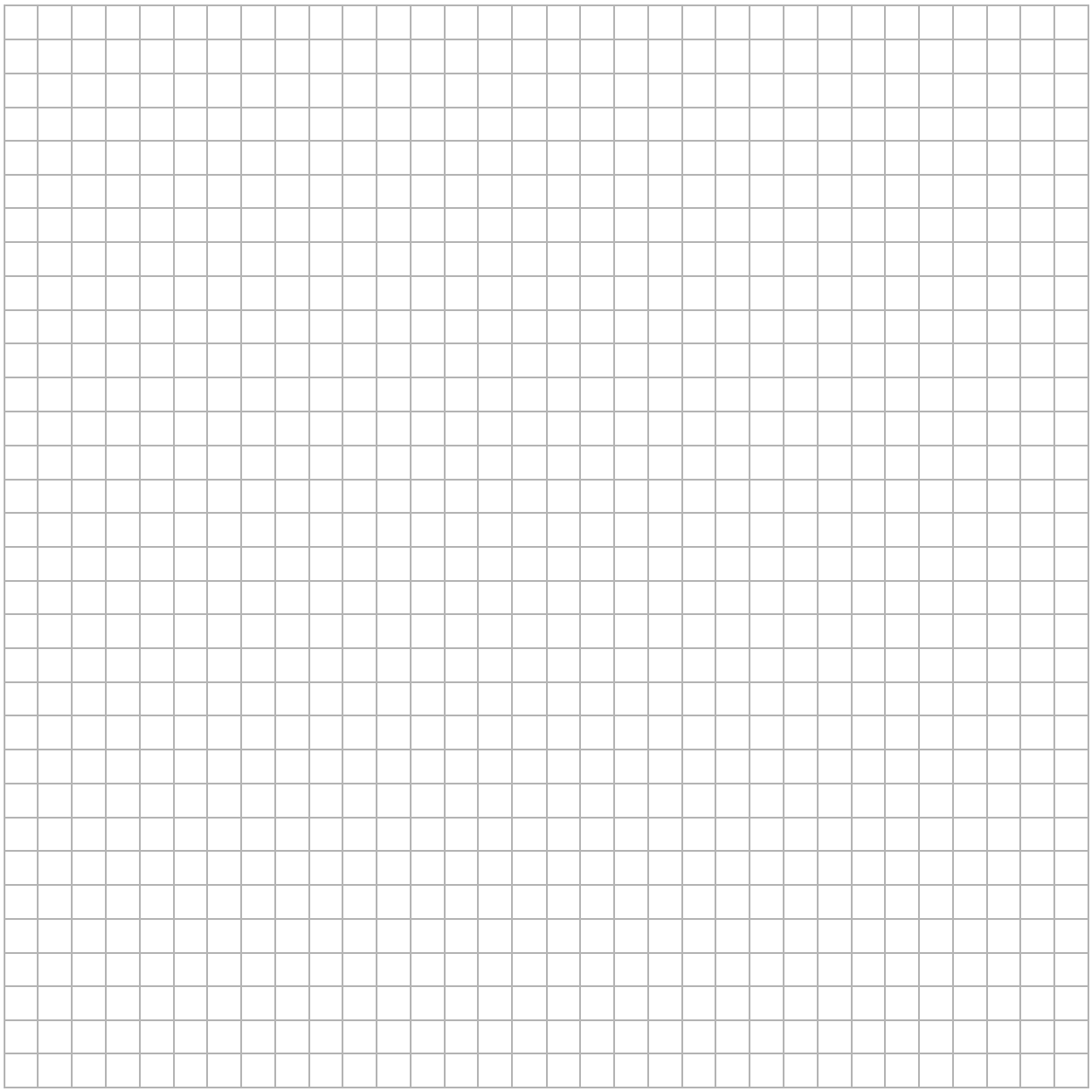
She is unable to make the measurements directly.



She will use triangles ADE and ACB.

Explain why, in geometric terms, triangles ADE and ACB are similar.

Create a ratio that can be used to find the distance *x* across the lake. Use this ratio and the measurements given in the diagram to calculate x, the distance across the lake.



Task

A surveyor wants to determine the distance across a lake.

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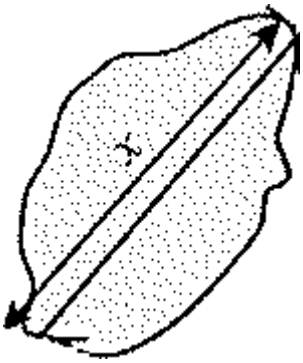
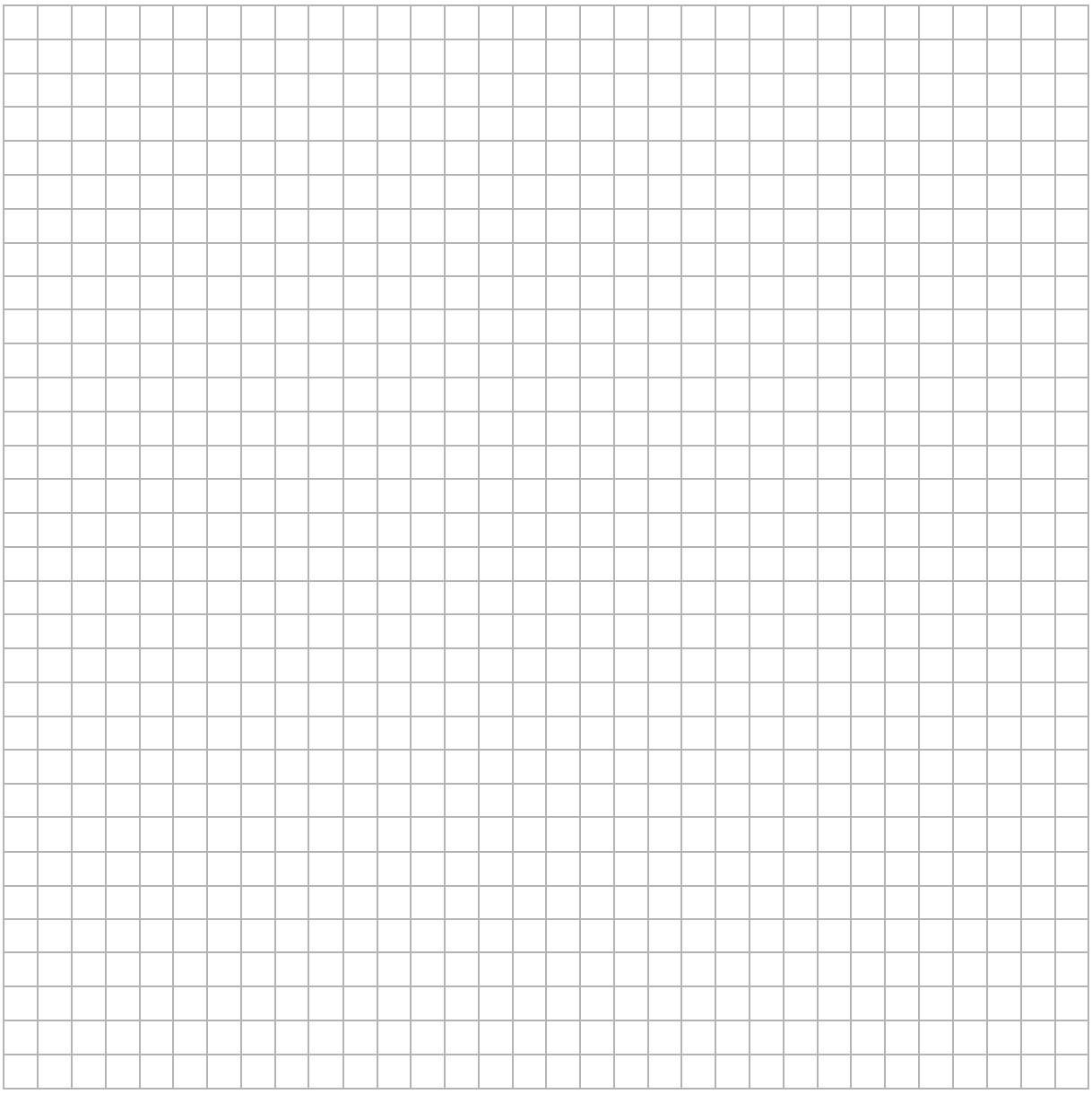


Suggest some measurements she could make and how she could use these to determine the distance across the lake.

Task

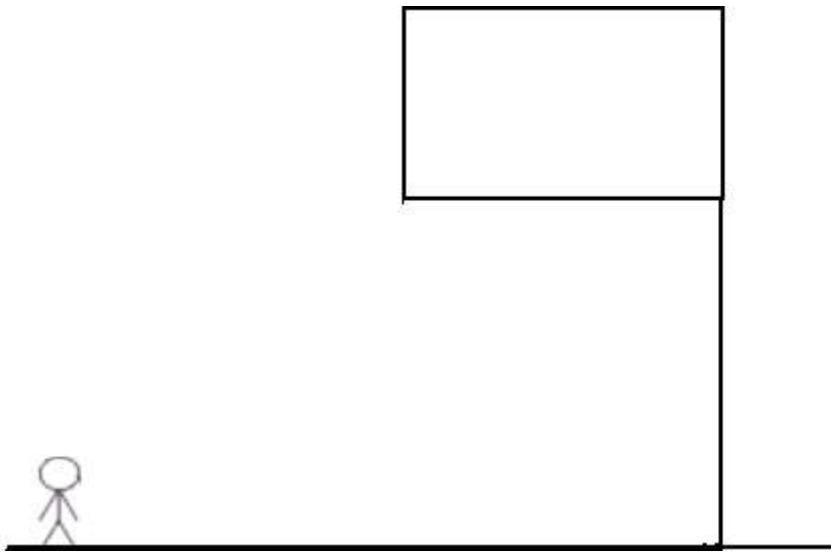
The JCDecaux advertising agency were looking for a building that was tall enough to accommodate an **18m** high rectangular billboard. An employee of the company thought he had found a building that would work. He is 2m tall and, on the morning he examined the building, he cast a shadow 0.5m long. The building cast a shadow 4m long.

Determine whether or not the building will accommodate the billboard.



Question **(Suggested maximum time: 8 minutes)**

(a) Some students were measuring the height of a flagpole near the school. They had a measuring tape and a **clinometer.**



The following measurements were taken

Height of student	1.5 m
Distance from Student to Flagpole	2 m
Angle of elevation of top of flagpole (θ)	

(a) Add these measurements to the diagram and show how the students could use them to calculate the height of the flagpole.

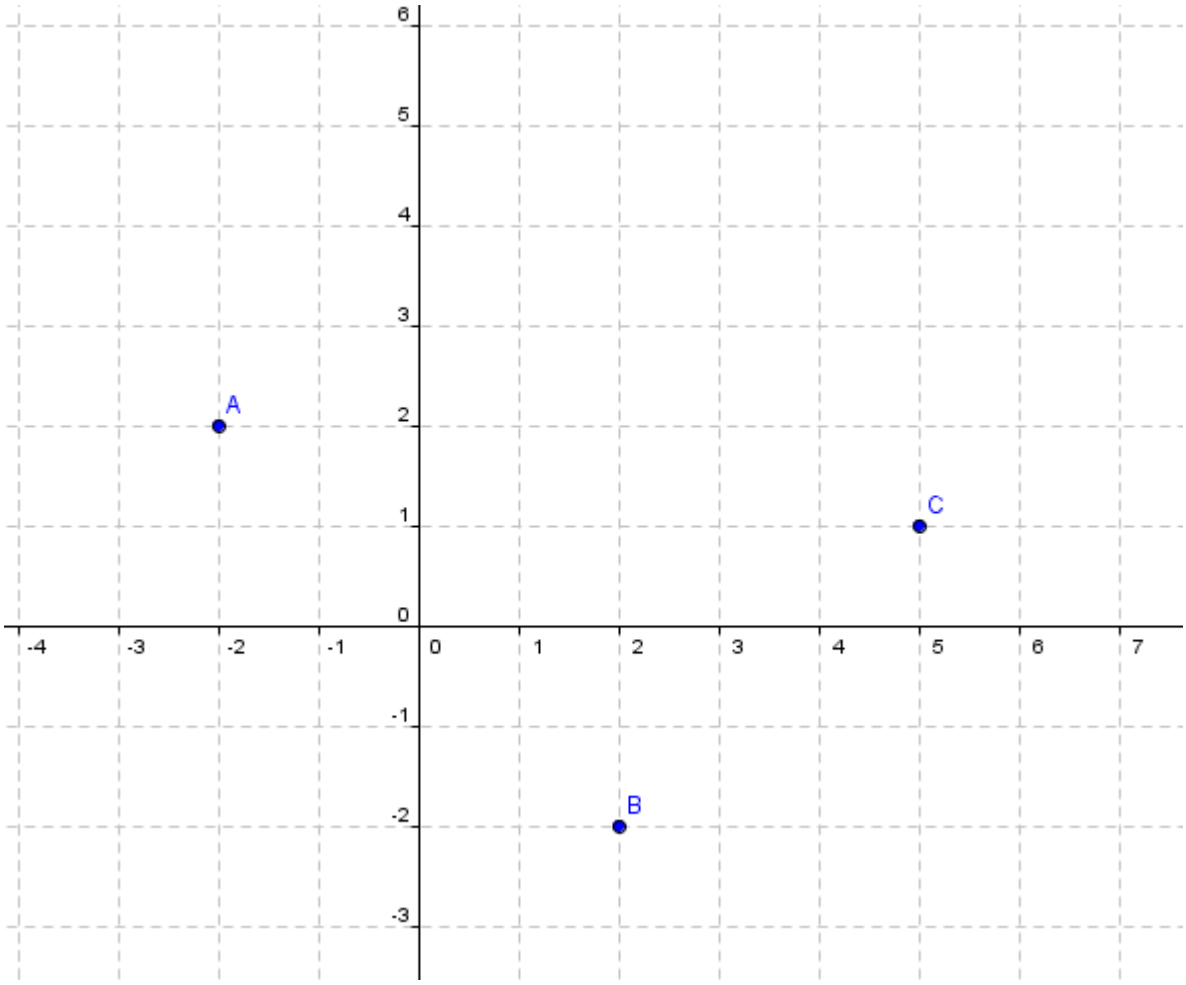
(b) The students calculated the height of the flag pole and found it to be 9.9 m. Unfortunately, before they could hand in their work, an ink blot spilled on it and covered the angle value. They did not want to go out and measure it again. Sophie suggested they work backwards to find the missing angle.

Find the missing angle by working backwards. You will need to use the table below.

Angle θ	**Tan θ**
38	.7813
37	.7536
36	.7265
35	.7002
34	.6745
33	.6494

Question **(Suggested maximum time: 8 minutes)**

Points A, B and C are marked on the coordinate grid below.



(a) Write down the coordinates of A, B and C

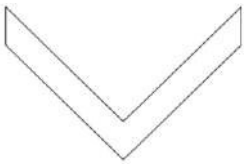
(b) Point D is such that $ABCD$ is a rectangle. Mark D accurately on the diagram.

(c) Write down a property of a rectangle.

(d) Show that the rectangle *ABCD* has the property that you wrote down in part **(c)**.

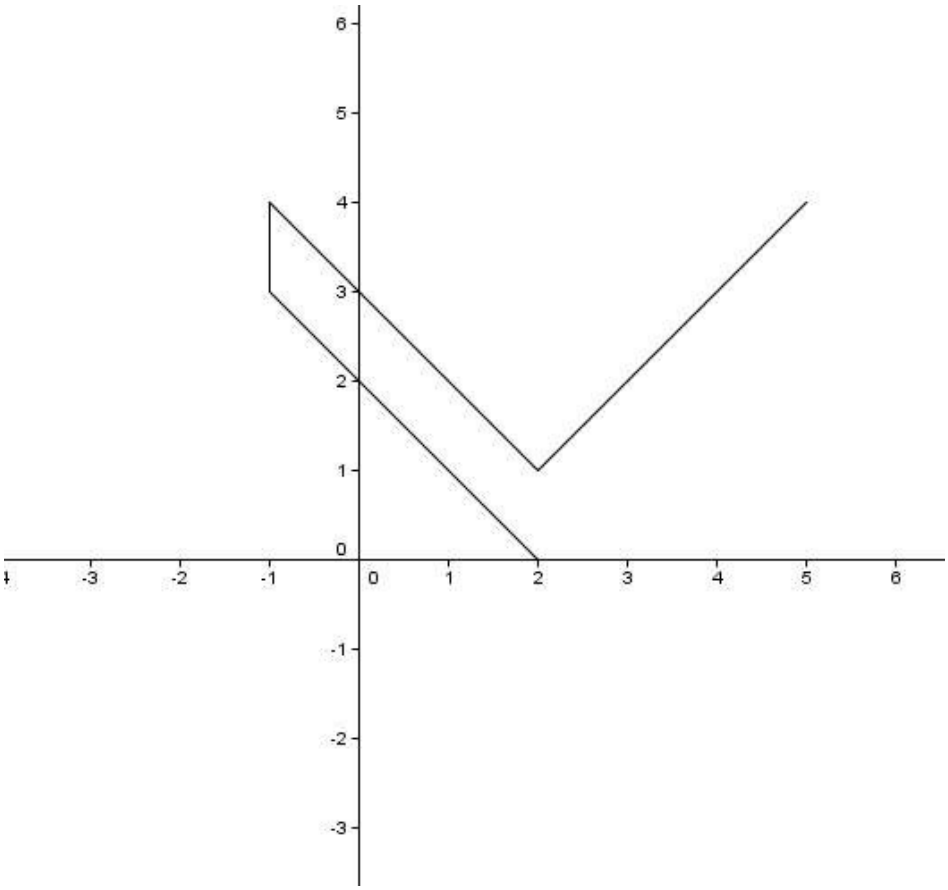
Question **(Suggested maximum time: 7 minutes)**

John is drawing plans for a logo. The logo is in the shape of the letter V as shown.



(a) On the diagram plot three points that John will need to join in order to complete the Logo.

(b) Draw the axis of symmetry on the logo.



Question **(Suggested maximum time 12 minutes)**

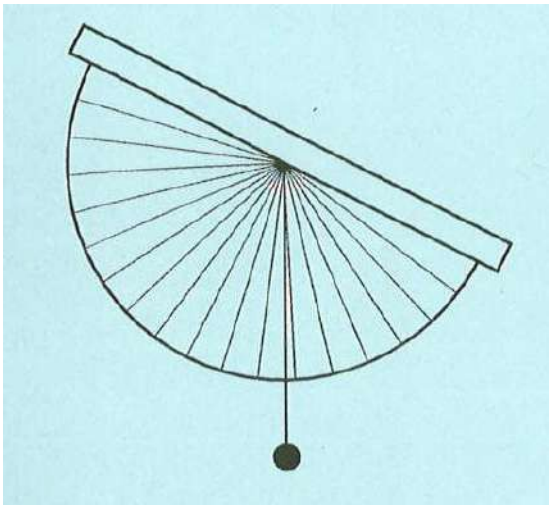
(a) Construct a triangle ABC, where $|AB| = 6$ cm $|AC| = 8$ cm and $|BC| = 10$ cm.

(b) What type of a triangle is this? Mathematically prove that this is so.

Question

The students mentioned in **(a)** above went to measure the height of the church.

(a) Peter explained his group's method:
"We made a clinometer from a protractor, a pen tube, some thread and a weight.
We measured the distance from here to the church and it was 92 metres.
We made sure the ground was flat,then we used the clinometer to look up at the top of the spire of the church. The weight had moved from 90 ° to 65 , so we knew the angle up was 25°. We worked out the height from that. But we had to remember to add on my height of 1.8 metres at the end."



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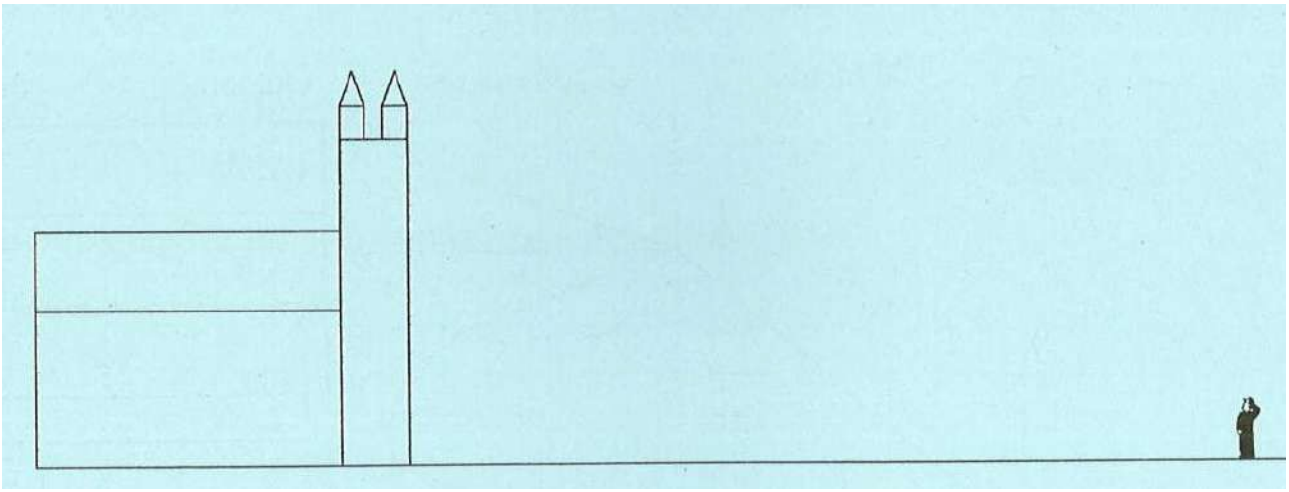
(i) On the diagram below, show the measurements that Peter's group made.



(ii) Show how Peter's group used these measurements to find the height of the church.

b) Hannah was in a different group from Peter. She explained her group's method for finding the height of the church:

"It was really sunny and we used the shadows cast by the sun.
Amy stood with her back to the sun and we used a tape measure to measure Amy's shadow along the ground from the tips of her toes to the top of her shadow's head. We also measured Amy's height and recorded the results in the table.

Then we recorded the length of the shadow cast by the church. We measured along the ground from the base of the church out to the end of its shadow and recorded this measurement."

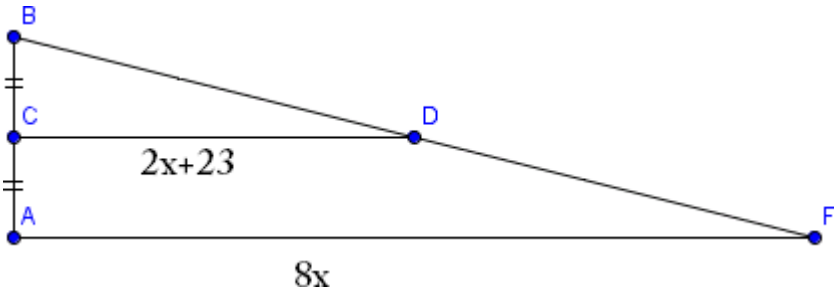
Amy's Shadow	2 m
Church's Shadow	69.4 m
Amy's Height	1.7 m

Show how Hannah's group used their results to calculate the height of the church.

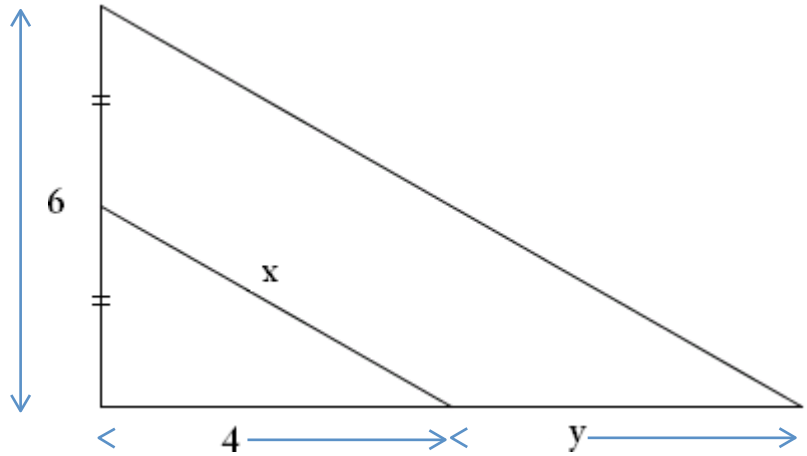
(c) The church is actually 50 metres high. Calculate the percentage error in each groups result.

Q. In the diagram, CD is parallel to AF and equal lengths are marked.

Find the value of x.

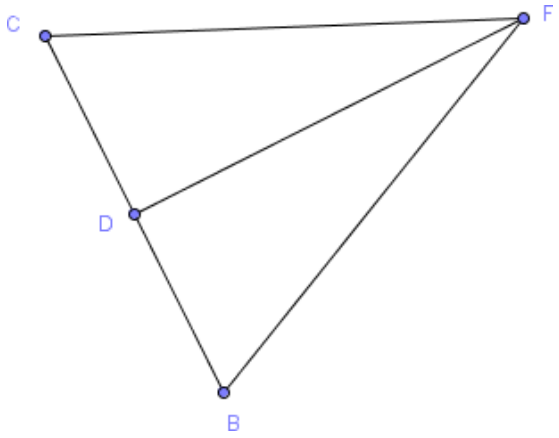


Q If the sloped lines are parallel, find the value of x and the value of y.



Q In triangle FCB |CD| = |DB| and |□FDC|= |□FDB| = 90°

Explain why the triangles FDC and FDB are congruent.

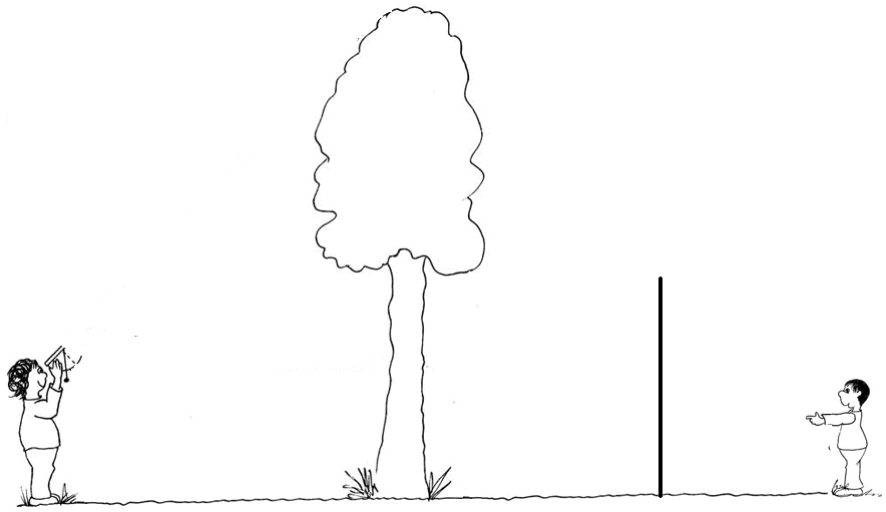


Q Jane and Stephen want to estimate the height of a tall tree which is vertical and stands on horizontal ground.

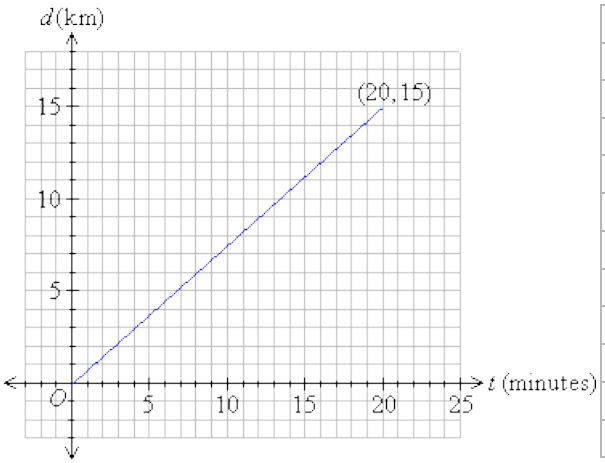
Jane has a **clinometer** and Stephen has a 100m measuring tape and a large **stake**.

Explain, using diagrams and your own reasonable measurements, how each of them can make an estimate of the tree's height.

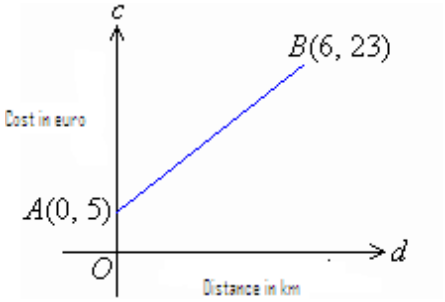
Account for any inaccuracies that might occur and suggest how you could minimise these inaccuracies.



Q (a) A cyclist travels for 20 minutes at a constant speed and covers a distance of 15 km, as shown in the diagram. Find the slope of the line and describe its meaning.



(b) The cost of transporting documents by courier can be represented by the following straight line



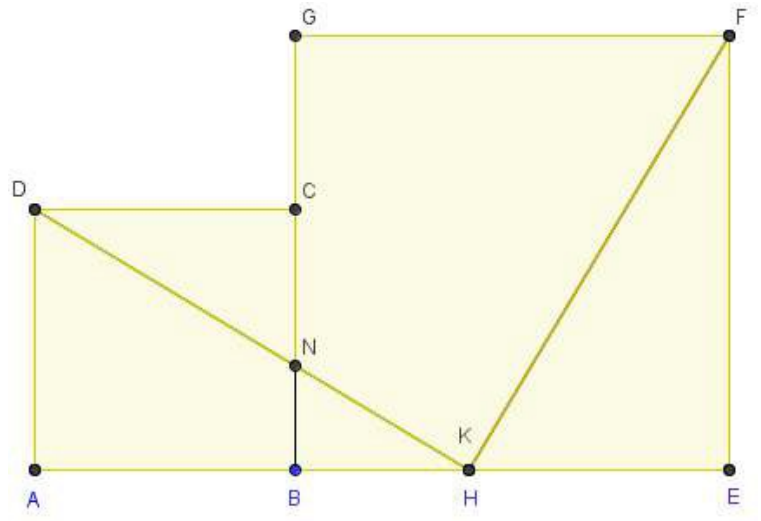
(i) What does each point represent?

(ii) Calculate the slope. What does this represent?

Q

The diagram (Fig. 1) shows two square tiles, ABCD and BEFG placed alongside each other. The point H is chosen along the side BE so that □HE□ = □AB□.

Fig. 1



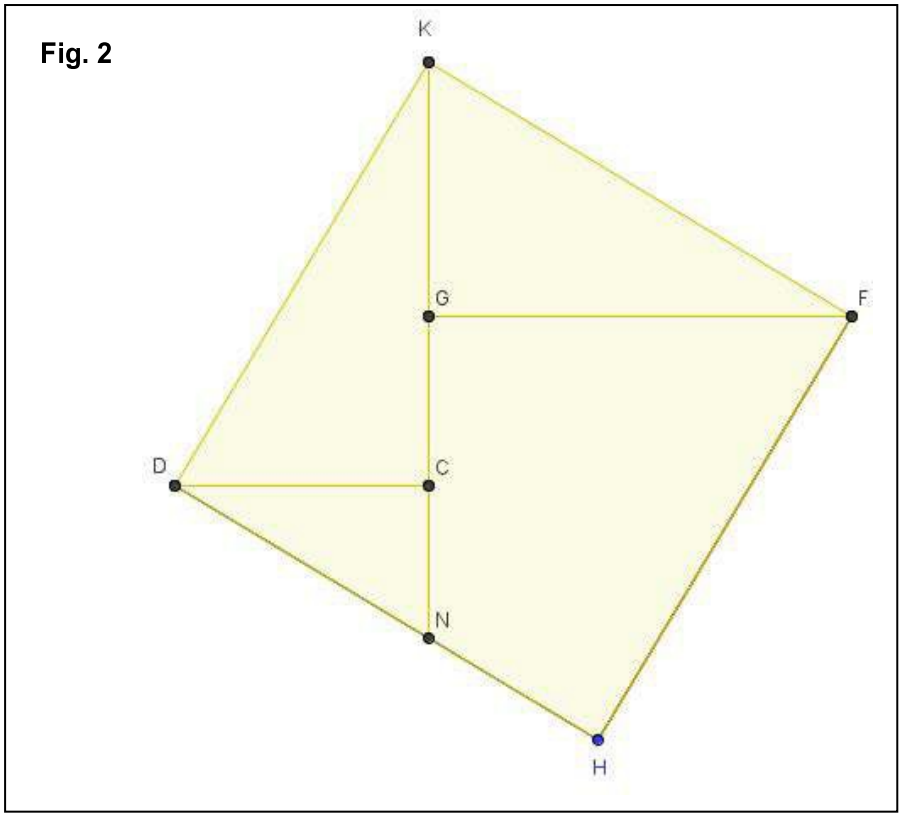
(i) Prove that the triangles DAH and HEF are congruent.

(ii) Prove that □ DHF is a right angle

The square tiles are cut along the lines DH and HF as shown and the pieces are moved so that ΔHEF lies in the position DCK and ΔDAH lies in the position KGF (see Fig. 2).

(iii) Prove that the new figure formed, DHFK, is a square.

Fig. 2

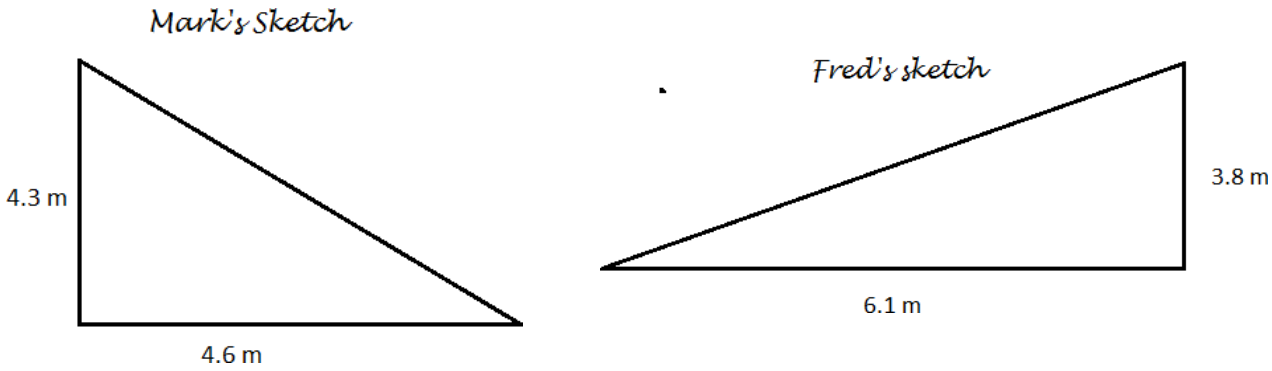


Q Trigonometry

Mark and Fred are designing a skateboard ramp. In *Skate Monthly*, they read the following advice

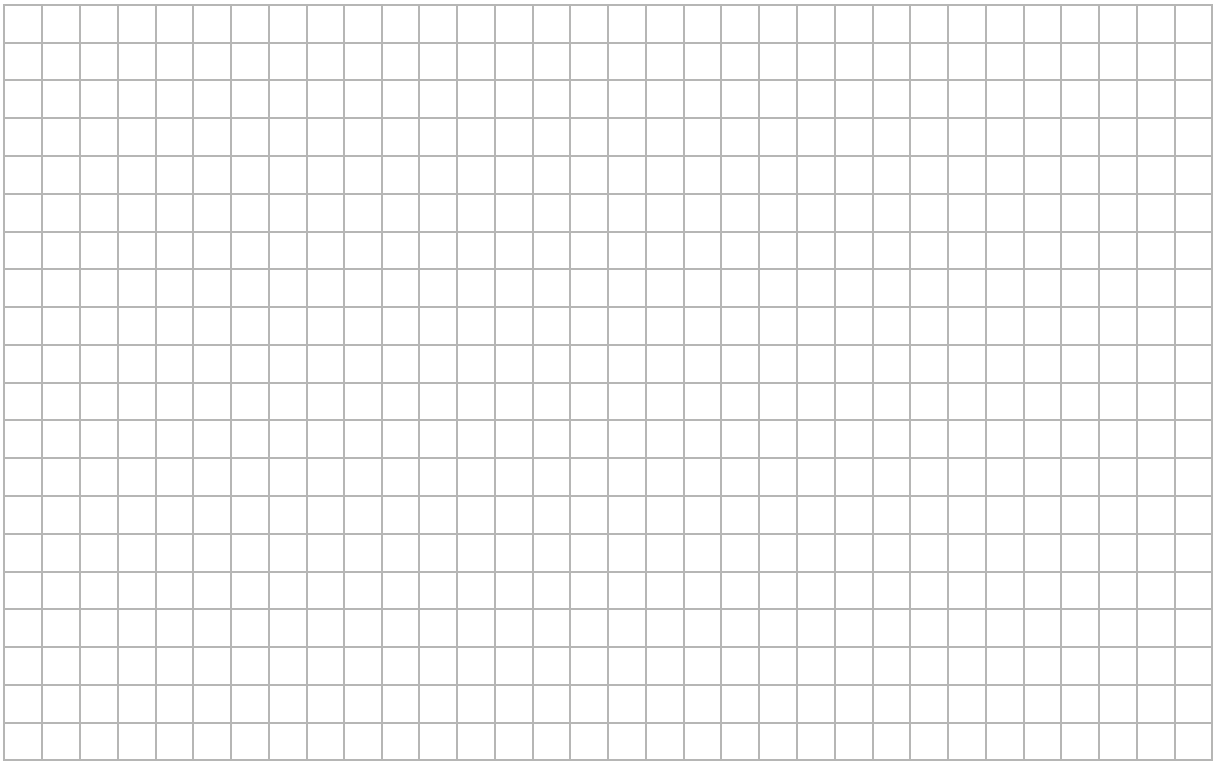
"to make a good skateboarding ramp, you need to find the balance between being too steep and too low. If it's too low, all you end up doing is getting a few inches off the ground, wiping out and looking silly. If it's too steep, you get halfway up, come back down, fall and look even sillier. It's best to keep the ramp angle with the ground between 30 and 45 degrees".

Here are Mark's and Fred's sketches:



(i) Use mathematics to decide which ramp is steeper (that is, has the greater slope).

(ii) Which ramp would ensure that the skater travels a greater distance on the ramp?



(iii) Does the angle which each ramp makes with the ground comply with the advice about angles given in *Skate Monthly*? Use mathematics to justify your conclusion.