Mathematical investigation: Chessboard Challenge

Learning outcomes in focus

Students should be able to:

U.4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations

U.7 make sense of a given problem, and if necessary, mathematise a situation

U.8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions

U.9 interpret their solution to a problem in terms of the original question

U.11 generate general mathematical statements or conjectures based on specific instances

U.13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely

AF.1 investigate patterns and relationships (linear, quadratic, doubling and tripling) in number, spatial patterns and real-world phenomena involving change

AF.2 investigate situations in which letters stand for quantities that are variable

AF.3 apply the properties of arithmetic operations and factorisation to generate equivalent expressions

Learning intentions

We are learning to:

- break problems down into parts
- investigate and extend a pattern in a systematic way
- represent pattern in tables, charts and graphs
- look for patterns and make conjectures
- validate our conjectures
- generalise our observations
- use words, tables, graphs and letters to communicate and justify mathematical ideas clearly

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Teaching and learning context

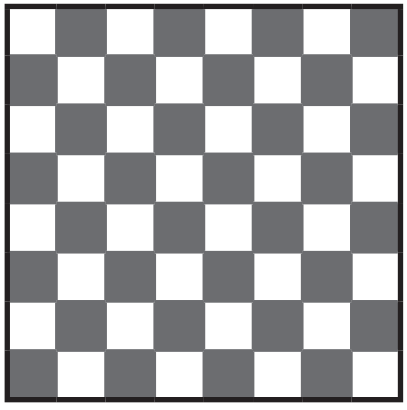
First year students were in the middle of a unit of learning which focused on investigating patterns in a systematic way. This task was used to introduce quadratic sequences. Their task required them to calculate the number of squares on a chessboard. Students recorded their results in tables which helped them spot patterns. They were reminded to record their findings clearly, to make and validate conjectures, to look for patterns and to try to generalise patterns they observed. To start the lesson, students used placemats to brainstorm how they might tackle the problem. We then discussed some strategies as a class. The students worked in groups to solve the problem during a 40 minute class and worked independently to create their speech for homework.

Task

You are a member of the English Debate team and your team is opposed to the following motion.

'*There are 64 squares on a chessboard.*'

Prepare a speech where you convince your classmates that this statement in wrong. Use tables, charts or graphs to support your arguments. Ensure that you explain clearly how many squares you think there are on a chessboard, and how you arrived at this conclusion.



Extension

Can you find a formula for the number of squares on an $n \times n$ grid.

Success Criteria

I can

SC1 extend a given pattern in a systematic way

SC2 record my observations in a table

SC3 make and verify conjectures

SC4 translate between different representations of a pattern

SC5 draw conclusions from my mathematical ideas

SC6 communicate my ideas clearly in words, tables, graphs or generalised expressions

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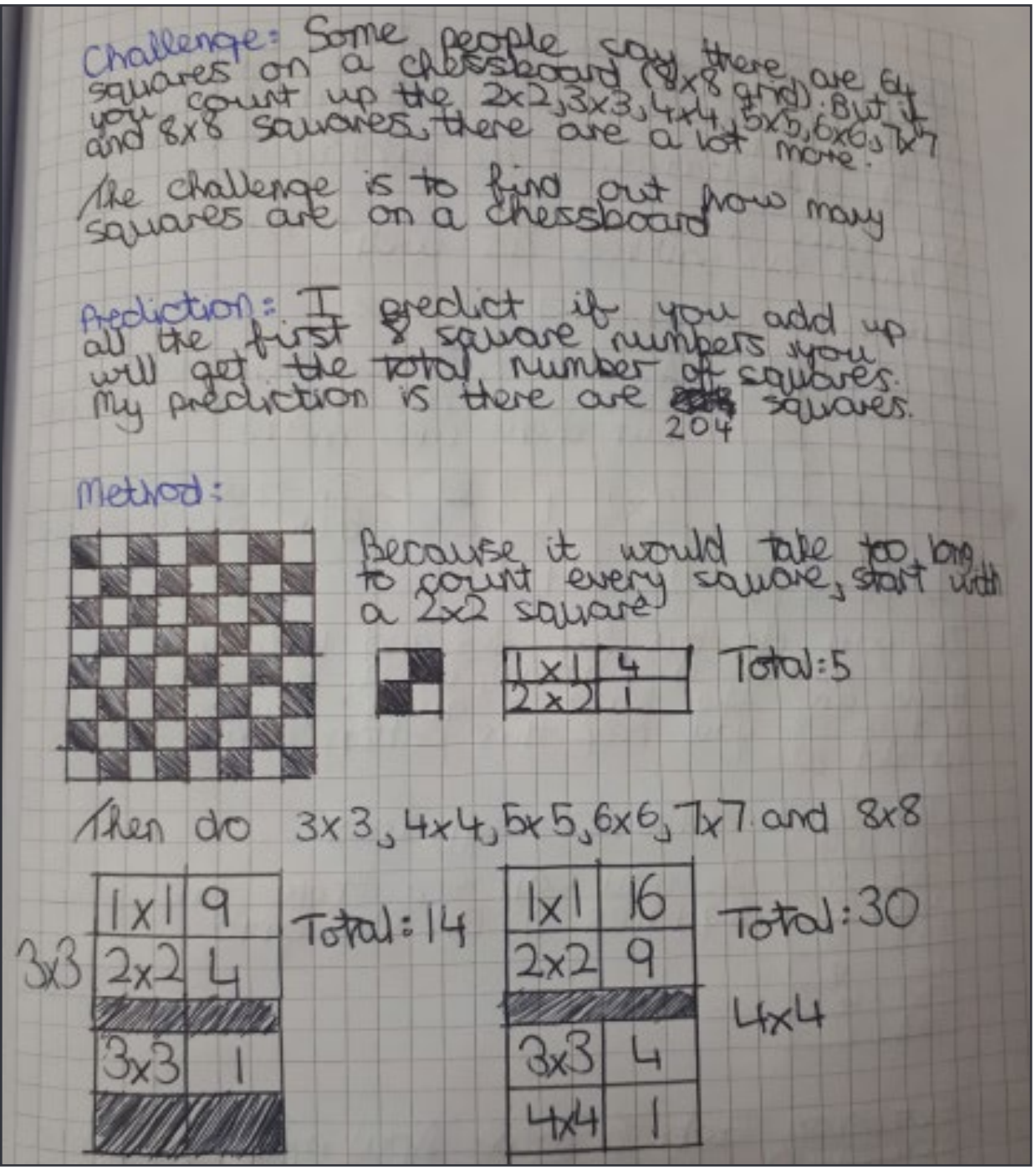

SC3: Makes a conjecture

SC1. Extends a pattern in a systematic way

SC2: Records observations in a table

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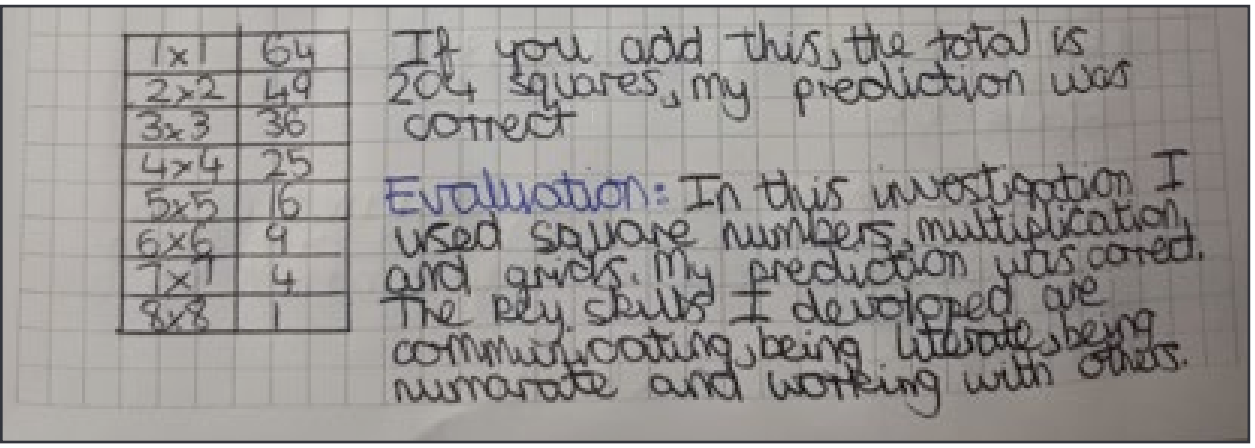
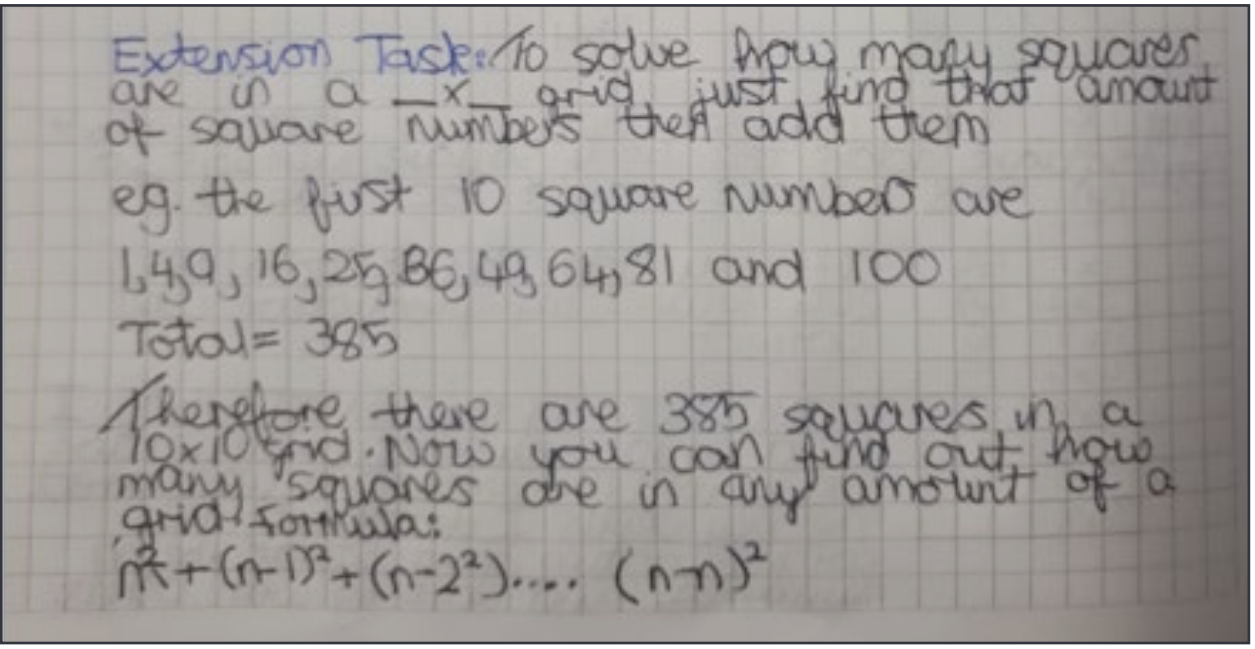


1x1	64
2x2	49
3x3	36
4x4	25
5x5	16
6x6	9
7x7	4
8x8	1

If you add this, the total is 204 squares, my prediction was correct

Evaluation: In this investigation I used square numbers, multiplication and grids. My prediction was correct. The key skills I developed are communicating, being literate, being numerate and working with others.



Extension Task: To solve how many squares are in a _x_ grid just find that amount of square numbers then add them

eg. the first 10 square numbers are

1, 4, 9, 16, 25, 36, 49, 64, 81 and 100

Total = 385

Therefore there are 385 squares in a 10x10 grid. Now you can find out how many squares are in any amount of a grid formula:

$n^2+(n-1)^2+(n-2^2)\ldots\ (n-n)^2$

SC3: Verifies conjecture

SC2: Records observations in a table

SC1. Extends a pattern in a systematic way

SC4. Translates between different representations of a pattern

SC6. Communicates ideas clearly in words and generalised expressions although there are errors in the mathematics

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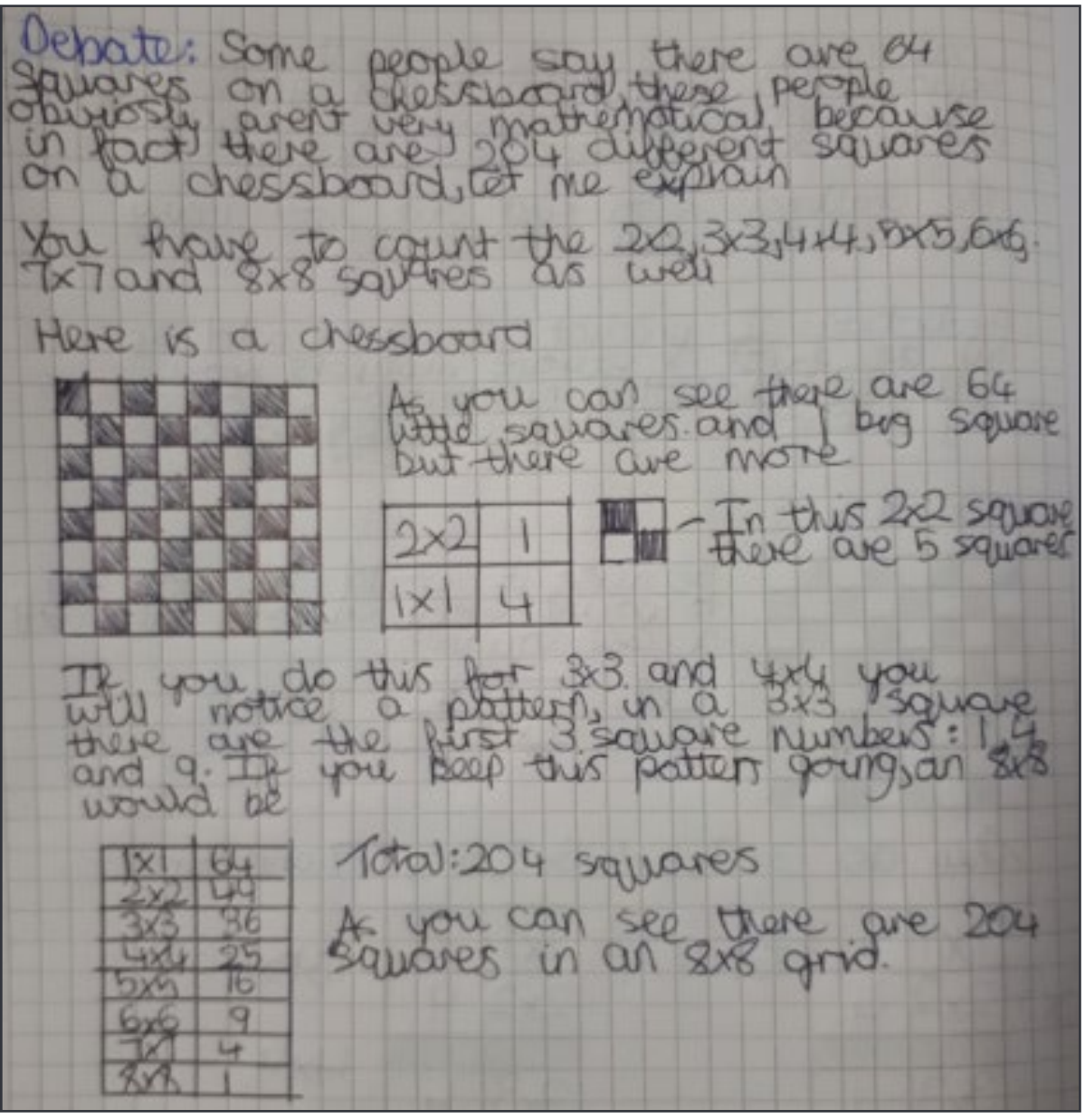

SC6. communicates ideas clearly in words, tables or generalised expressions

SC4: Translates between different representations of a pattern

SC2: Records observations in a table

SC5. draws conclusions from mathematical ideas

Overall judgement: Above expectations

Exceptional | Above expectations | In line with expectations | Yet to meet expectations