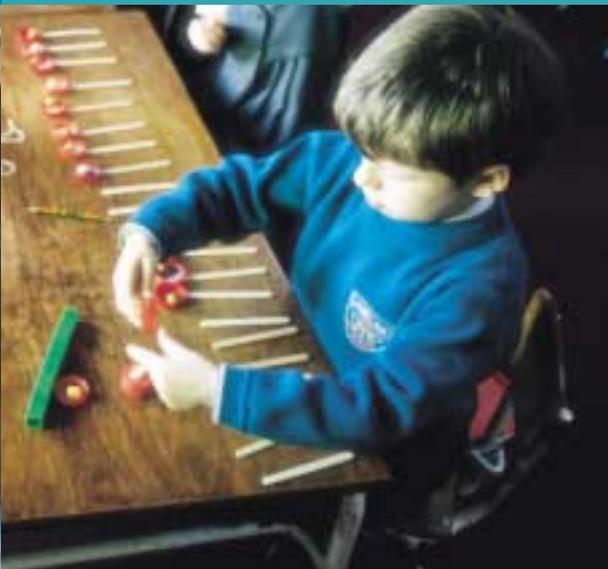


Primary School  
Curriculum



# Mathematics

Teacher Guidelines



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# Mathematics

Teacher Guidelines

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*Children learn from  
the people and  
materials around them*

Section 1

# Mathematics in the primary curriculum

# Mathematics in the primary curriculum

## Introduction

Mathematics is recognised as one of the sciences and has been described and defined in many different ways. It is a creative activity and is one of the most useful, fascinating and stimulating divisions of human knowledge. It is a process of managing and communicating information and has the power to predict and provide solutions to practical problems as well as enabling the individual to create new imaginative worlds to explore. We use mathematics in everyday life, in science, in industry, in business and in our free time.

Mathematics education is concerned with the acquisition, understanding and application of skills. Mathematical literacy is of central importance in providing the child with the necessary skills to live a full life as a child and later as an adult. Society needs people who can think and communicate quantitatively and who can recognise situations where mathematics can be applied to solve problems. It is necessary to make sense of data encountered in the media, to be competent in terms of vocational mathematical literacy and to use appropriate technology to support such applications. This curriculum will be a key factor in preparing children to meet the demands of the twenty-first century.

Mathematical applications are required in many subjects. The ability to interpret and handle data is of particular relevance in history and geography. *Measures* and *Shape and space* relate to the visual arts, physical education and geography. Integration gives the child a reason and motivation to develop mathematical skills and concepts that can be used in all subjects. Numeracy and estimation are particular to mathematics, while evaluating findings, reporting back, predicting and reasoning are used both in mathematics and throughout the curriculum.

### A constructivist approach

One procedure would be as follows:

- children discuss the problem
- try a possible approach
- further discussion
- modify arising from the interaction
- construct concepts from deductions
- arrive at a solution or solutions
- discuss results
- record

## Mathematics in a child-centred curriculum

The child learns from the people and materials around him/her. It is experience of the social and physical world that is the source of concepts, ideas, facts and skills. Integration of these experiences is the vital ingredient. If the child is given the chance to manipulate, touch and see objects that help him/her to acquire an understanding of concepts, he/she will understand more effectively than if words and symbols are the only learning tools. Often discovery learning alone will not be enough. The child needs guidance in formulating theories about what it is he/she is discovering. The child also needs help in developing the language for describing accurately what it is he/she is doing. The teacher and the child's peers have a vital role to play in his/her educational experience. Yet ultimately it is the child who creates the balance between his/her knowledge and the knowledge of those around him/her.

## Constructivism

Constructivist approaches are central to this mathematics curriculum. To learn mathematics children must construct their own internal structures. As in reading and writing, children invent their own procedures. We accept that children must go through the invented spelling stage before they begin to develop a concept of the structures of spelling. The same is true of mathematics. Young children attempt to count or order things in the environment and they develop rules for themselves to do so. They should be encouraged to try out these personal strategies, to refine them by discussion and to engage in a wide variety of tasks.

It is in the interpersonal domain that children can test the ideas they have constructed and modify them as a result of this interaction. When working in a constructivist way children usually operate in pairs or small groups to solve problems co-operatively. Tasks that are written on one sheet can be given to groups of two or more children. This makes consultation, discussion and co-operation essential. Children work at their own pace but are encouraged to complete the task as fully as possible within the set time. They are expected to respect one another's solutions, not to discredit partners' reasoning, and to discuss the train of thought used in the process.

This sociocultural theory sees cognitive development as a product of social interaction between partners who solve problems together. It acknowledges the importance of the home and family in the child's learning and focuses on group interaction. It is a process approach rather than a step-like, incremental one. One form of instruction used is *scaffolding*. Here the teacher modifies the amount of support according to the needs of the child by modelling the behaviour, for example possible methods of approaching a problem. The teacher breaks down the task and makes the task manageable for the individual child, thus supporting the development of the child's own problem-solving skills.

Through discussion the child becomes aware of the characteristics of a task. He/she must be encouraged to use the correct vocabulary needed for a particular task. Young children are egocentric, and it is through social interaction that they can begin to appreciate the points of view of other people. Sequences of instruction involve discussion, hands-on experience and practical exploration.

As adults we expect objects to behave in a stable and predictable manner. Children have to learn to recognise these attributes. They need to handle and use a variety of objects in order to form their own rules and structures for dealing with the world. This is of particular importance in mathematics.

Children need to work out *when* to use a particular plan, *what* they want to achieve and the actual *procedure* needed to complete the task. Through experiencing many different types of problems they become more efficient. The wider the range of problems they encounter the more likely they are to generalise the rules and use them in new situations.

While direct instruction is very important in mathematics, children also need to develop their own learning strategies. We need to teach children to look at *how* they arrived at a result rather than just concentrating on the answer as an end in itself.

Children need training in the skills of collaboration and co-operation, in listening to, accepting and evaluating the views of others. These skills are applicable throughout the curriculum. Work on open-ended problems, where the emphasis is placed on using skills and discussion rather than seeking a unique solution, is recommended. Many methods may be used in solving a mathematical task.





# The content of the mathematics curriculum

# The content of the mathematics curriculum

## Strands of the mathematics curriculum

- Number
- Algebra
- Shape and space
- Measures
- Data

## Content for first class

Strand: Number

Strand unit: Place value

### Content

- explore, identify and record place value (0-99)

### Exemplars:

*rename groups of counters as units or as tens and units*

## Structure of the curriculum

The areas of content in this mathematics curriculum are referred to as strands. The strands form a network of related and interdependent units. These are further developed as strand units. Each strand unit contains the content and some exemplars for that unit. *Measures, Shape and space, Data* and *Algebra* form a greater part of the curriculum than *Number* but number is an integral component of all of the strands. No strand stands alone and this should be reflected in timetabling.

## How the content is presented

- content is presented in two-year blocks, for example first class on the left and second class on the right of each page. The treatment of content is common to both classes. This presentation helps the teacher in planning and in revision
- the strand unit is in coloured type to the left of the page
- the content statement is indicated by a bullet
- the content of the exemplars is in italic type. These are limited suggestions for implementing the strand unit. It is envisaged that teachers will develop and extend these suggestions as they work through the programme

- vocabulary relevant to the strand unit is shown in bold type, for example **long/longer, short/shorter, positive/negative**
- the sequence of presentation of the strands in the content document is: *Number, Algebra, Shape and space, Measures* and *Data*. This does not imply a hierarchy. Strands can be taught in parallel rather than one after the other and this facilitates the use of aspects of number throughout the mathematics curriculum. This is called *linkage*; for example, in teaching fractions it is possible to link with the strand *Shape and space*
- integration opportunities are indicated in some strand units but these are merely suggestions. It is hoped that teachers will identify other opportunities to integrate mathematics with the rest of the curriculum.

## Early mathematical activities

### Strand units:

- Classifying
- Matching
- Comparing
- Ordering

## One example of the ceiling on number work:

### Junior infants

- counting 1-10
- comparing, ordering, combining and partitioning sets of objects, 1-5
- reading, writing and ordering numerals, 1-5

## Strand content

### Early mathematical activities

All children come to school with some mathematical knowledge and language, which they have gained at home and through play. It is through play that young children learn to share ideas and co-operate, to share toys and express ideas. This differs greatly from child to child. Play in the classroom develops these skills by providing structured situations for the child to explore.

Early mathematical activities guide this basic knowledge and provide a solid foundation for subsequent mathematical investigations both at school and at home. It is envisaged that this strand will be covered intensively and will therefore provide a solid basis for subsequent work. The emphasis will be on developing the use of correct mathematical language and confidence in handling the materials.

## Number

A ceiling has been placed on number work to allow for more extensive treatment of the mathematics programme as a whole and to allow more time for concept development. Less emphasis is placed on complex computational exercises involving whole numbers and fractions. It is hoped that writing will not be the sole method of recording and that symbols will only be used when appropriate to the child's level of understanding. Recording can also be concrete, oral, pictorial or diagrammatic or can include model-making. Calculators have been introduced from fourth to sixth class when it is expected that the children will have achieved mastery of basic number facts.

All number work should be based as much as possible on the children's own experiences and real-life examples used. It is essential that children see mathematics as relevant to their own lives. Projects that use advertising features from newspapers or prices from local shops and supermarkets can be used to link many of the strands.

Work on fractions and decimals in general will place more emphasis on understanding the relationships between them. This might best be achieved by the use of manipulatives and paper-folding.



In the addition and subtraction of simple fractions and simple mixed numbers it is suggested that equivalent fractions be used to simplify calculations, for example

$$\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8}$$

Children will need experience of many examples of equivalent fractions, and this is best done through generating families of fractions, for example

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

The same process can be followed with thirds, fourths and fifths.

$\frac{1}{3}$  belongs to the set of equivalent fractions

$$\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12} \dots$$

$\frac{1}{4}$  belongs to the set of equivalent fractions

$$\frac{1}{4}, \frac{2}{8}, \frac{3}{12} \dots$$

so

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

In sixth class, listing multiples rather than constructing factor trees should be used to find common multiples and denominators.

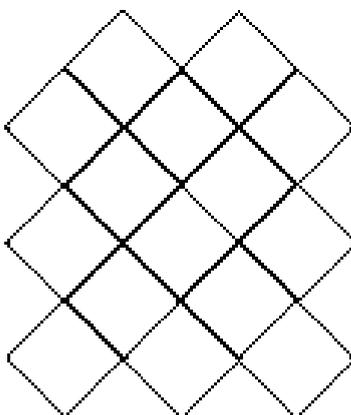
Initial examples should be restricted to simple fractions that are in common use. Hundredths and thousandths are best introduced through money and measures and their relationship with decimal fractions.

## Algebra

Algebra, which includes patterns, sequences and statements such as  $2 + \square = 5$ , has always been part of the curriculum but is now formally recognised at all levels. At infant level it includes copying and extending patterns in colour, shape, size and number. First and second class progress to exploring and using addition facts, and third and fourth class work includes exploration, extension and description of sequences of numbers. At the senior level positive and negative numbers are introduced. Rules and properties of brackets, priority of operations, equations and variables are also covered. It is expected that the numbers used will be kept small so that the children can understand the concepts presented.

## Shape and space

This strand explores spatial awareness and its application to real-life situations. Here again the child's experience must be a practical one. The child must know how to choose the correct mathematical tools for the problem and be able to use the correct vocabulary to describe his/her work. *Shape and space* is particularly suited to integration, as children will enjoy finding shapes and angles in the environment and creating tessellating patterns in art. Since angles and tessellation are so closely related it will be necessary to encourage children to make this connection when covering surfaces and examining paving and tiling. This leads to the discovery that shapes that tessellate form an angle of  $360^\circ$  at the points of contact.





## Strand: Measures

### Strand units:

- Length
- Area
- Weight
- Capacity
- Time
- Money

## Measures

This strand has six strand units. In this strand children are given frequent opportunities to undertake practical activities. These activities are particularly useful in facilitating linkage within the strand units. Fractions, decimals, percentages and operations can be applied in measuring activities.

Children learn how to select the correct measuring instruments and the most appropriate strategy for tackling a particular problem. Problems set in these strands should be mainly practical, with the totals easily verified by measuring. Initially children work with the measuring unit present, i.e. the metre stick, half-metre measure or centimetre. This provides a visual reference when estimating and assists in the development of the concept of a metre or centimetre.

Children should be taught from an early age to estimate the weight, length or capacity of an object. This can be done by comparison, using a labelled object that weighs one kilogram, for example a bag of sugar. The child can feel the weight of the labelled object and compare it with the test object by hand weighing. The actual weight can be discovered by using a weighing scales.

Children will need to handle and use a wide variety of materials within each strand. They will need to investigate the capacity of tall, narrow containers and short, wide containers. They can then discuss and compare their findings. Activities in weight can include the comparison of the weight of a small, heavy ball of Plasticine with the weight of a large bag of polystyrene.

When learning about capacity they will need to become familiar with the major and minor markings on containers and note the differences between the various methods of marking.

The reasons for using standard measuring instruments should be explored in a practical way with the child; would we all get the same amount if we bought potatoes by the bucketful? Children who have grasped the concept of standard measure will realise that there could be many different sizes and shapes of bucket. Teacher observation of such learning experiences can be valuable in assessing the child's progress.

### *Introduction of the euro*

The euro coinage will be introduced on 1 January 2002. Companies can use it for paper transactions from 1 January 1999 but the coinage will not be available until 2002. In the revised curriculum we have provided alternative objectives and exemplars which are to be used after the introduction of the new currency, and these can be seen in the strand units on money at each level.

### **Data**

Graphical representation and interpretation have always been part of the curriculum, but data handling is now a separate strand. Infant classes collect personal information and represent it on a pictogram; older children create and interpret bar charts and pie charts. Interpreting and understanding visual representation is essential, as the child needs to be enabled to interpret data in an increasingly technological world, and it is hoped that, where available, information technology will be used by children in data-handling exercises.

We manipulate data in the formulation of simple bar charts and pictograms at quite an early age. Databases allow us to extend this knowledge to the real world by handling larger amounts of information. However, children must understand how important it is to enter relevant data and ask clear questions if the information we extract from the database is to be of any use. In collecting information on, for example birds, the children will have to decide under which headings they will collect the information: name of bird, habitat, number of eggs per clutch, wing span, main food source. There can be much discussion on what they wish to discover: for example, do all fish-eating birds live near the sea? Do birds with a large wing span lay more eggs? By the time they have collected the information, constructed the headings, entered the data and considered some questions they will already have gained an understanding of how a database works. Using the database to find the answers then becomes a technical activity, merely a tool for the manipulation of a large amount of information.

### Examples of activities using chance

- what will happen next?
- record outcomes of multiple dice throws

The concept of *chance* has great importance in a number of areas. We have all experienced chance when playing board games and participating in sport. It is fundamental to decisions made in such areas as business and weather forecasting. It represents real-life mathematics and promotes thinking and discussion. Many children and adults have poor or inadequate notions of chance and likelihood. Some assume there are 'lucky' numbers, which

turn up more often than other numbers in a raffle. This can provide the basis for interesting experiments. These topics are introduced through problems, practical experiments and simulations that help to develop intuitive foundations for future work and are fun for the child. It is intended that the area of chance be explored in a practical manner with the emphasis on accurate use of language.





# School planning for mathematics

# School planning for mathematics

## The school plan should consider:

- the particular needs of the school community
- teaching materials  
equipment  
textbooks and worksheets  
calculators
- mathematical language and methodologies
- assessment and record-keeping

## Curriculum planning

A school plan should be comprehensive and balanced and reflect the particular needs of the school community. The implementation of this mathematics curriculum will require a review of the school plan at present in operation in schools. It will be necessary to determine how the school intends to incorporate the new aspects into its existing plan.

## Teaching materials

Teaching materials will be required at all class levels and in every strand. As children develop their individual learning styles it is important that they have experience of a variety of materials and the freedom to choose from these when exploring a mathematical task.

### *Equipment*

There should be enough equipment for the use of at least one group of children in the classroom. If children are to learn from *doing* they must *do*. A list of some mathematical equipment is given at the end of this document. Children can collect objects for counting and sorting, and these can be kept in a tray on a desk for use by a group or individual child as required.

### *Textbooks and worksheets*

The use of teacher-developed worksheets is important. This encourages teachers to share expertise and adapt work to the needs of a particular class or school. Textbooks can be evaluated by the staff, and consideration should be given to

how they reflect the objectives of the curriculum. In general, textbooks should include a balanced treatment of all the strands, varied presentation of problems and an emphasis on the use of manipulatives. They should encourage investigation and provide the child with structured opportunities to engage in exploratory activities. A variety of textbooks could be made available to the children based on the quality of their content in particular strands.

To implement this curriculum it will be necessary to present work to the children in a wide variety of ways. Having a selection of different mathematics books in the classroom or pockets of graded worksheets can help in providing extension work for children who have mastered a concept. Giving the children 'clue sheets' for approaching a problem helps them to work more independently.

### *Calculators and computers*

Calculators and computers can enhance the implementation of this curriculum. Calculators are part of the programme from fourth to sixth class. Schools will need to have a common policy on the type of calculator used in the school, the quantity required and how they are allocated.

## Mathematical language and methodologies

A common teaching approach to areas of difficulty such as subtraction, multiplication or fractions is very important in ensuring continuity and consistency. A school policy on such areas can be of great assistance to teachers transferring from one class to another or to long-term substitutes. This policy can be communicated to parents so that they can help children constructively with homework. Mathematics is a highly structured subject, and it is essential to have a solid foundation before moving from one level to another. Extension work on a unit or throughout several units is encouraged rather than a strictly vertical progression through a topic. Using mathematical skills in a variety of ways helps reinforcement, as the child can see the reason for the transfer of those skills.

Mathematics can be viewed as a language in itself with its own vocabulary and grammar. It must be spoken before being read and read before being written. Some everyday words take on new meanings when used in mathematics and can cause confusion for children, for example *odd*, *count* and *difference*. It is important that the school as a whole encourages the accurate and effective use of mathematical language.

## Assessment and record-keeping

Assessment and record-keeping are important factors in documenting the continuity of a child's progress. *All* the strands of the mathematics programme should be assessed.

Schools should have a common policy on the format of and terminology used in record sheets. In addressing individual differences the school must have a policy on assessment, remediation and referral.

The manageability of assessment tools and the storage of information should be addressed at school level. Information arising from assessment must be treated with confidentiality. Consideration should also be given to the frequency of delivery of tests.

Assessment can help the teacher and the child in a number of ways:

- it can show how children are progressing in the different strands of the mathematics curriculum and therefore provide a basis for planning future work in those areas
- it can help in the pacing of work by indicating the strengths and weaknesses of a particular group or indicate their readiness to proceed to a new topic
- it can have a diagnostic role in identifying particular areas of difficulty for a child who may need extra help.

Encouraging children to use self-assessment techniques can enhance their enjoyment of the subject and make them more active participants in their learning. This can be achieved by asking them to look at how they arrived at a solution and verbalising how they can use this information in the future.

### Organisational planning should consider:

- resources
- homework
- home-school links

## Organisational planning

Organisational planning in mathematics involves the whole school community. Planning should consider, for example resource requirements, homework policy and home-school links. Consultation with parents and the board of management is important for the successful implementation of this curriculum, as is discussion of issues that arise from curriculum planning in mathematics. Such planning should contribute to the overall school plan which will be reviewed by the board of management. The board, within the resources available to it, will provide support for the development and implementation of the school plan.

### Resources

Access to an adequate supply of suitable teaching materials is essential for the development of an investigative approach to the teaching of mathematics. Decisions will have to be made about the purchase, storage and availability of equipment.

Some class levels will require daily access to certain materials, while other items may only be required on an occasional basis. It is important that access to such equipment is clearly defined so that maximum use can be made of it. It is also useful to have a policy on repair or replacement of damaged materials.

The staff could decide to have equipment stored centrally by class level or by strand, for example *Measures*, or *Shape and space*. Useful worksheets or workbooks on a topic could also be collected and stored with the equipment.

### Homework

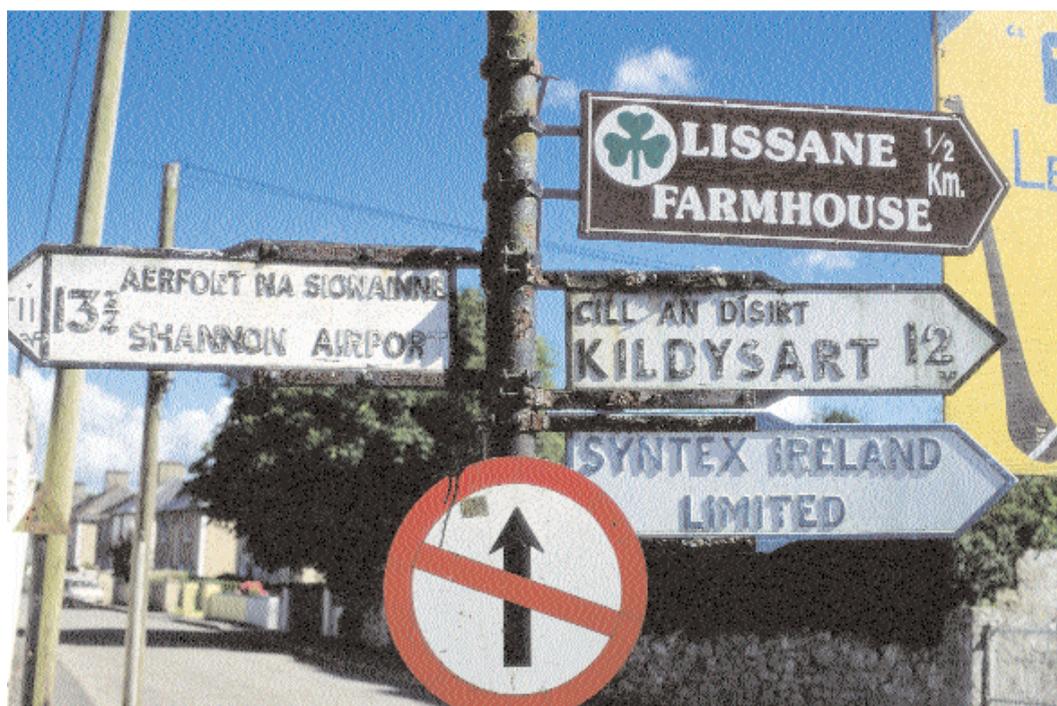
Homework should be seen as reinforcement, as it offers an opportunity to widen experiences begun in the classroom, for example work with capacity or finding the area of a room. It encourages organisational skills and the ability to work independently.

In developing a school plan consideration must be given to a policy on homework. Homework creates a link between home and school. It is important to communicate with parents about the correct terminology and methods being used by the children and to make homework assignments realistic, practical and relevant. Teachers can set alternative forms of homework, for example research in the local library or using measuring skills in cookery.

## Home-school links

Communication between teachers and parents about the content of the mathematics programme and the methods being used is important. Homework activities can be suggested and the importance of the child's readiness for new topics explained. Parents can help their children informally by encouraging the correct use of mathematical language and the use of number, estimation and mental strategies in everyday life. The importance for young children of play and exploration with, for example sand, water, bricks or blocks can be highlighted.

Parents can provide useful information for the teacher about the child's early number or mathematical experience. They will also be able to help the teacher's understanding of the child's attitudes to mathematics and his/her use of mathematics in daily life.





# Classroom planning for mathematics

# Classroom planning for mathematics

*A well-planned, well-equipped, easily accessible mathematical work-place promotes enthusiasm, curiosity and creativity in the child.*

## Classroom organisation

### Classroom management

The classroom is the work-place of both teachers and children, and a well-managed work-place increases job satisfaction and enhances the learning process. Planning facilitates co-operation and the best use of resources and space. This is particularly relevant when mathematics is being integrated with other subjects. Integration with geography may require the use of maps or globes. Science equipment may be required for work on capacity. Integrating mathematics with other areas of the curriculum enables children to use mathematics in a meaningful way.

### The mathematics area

Ideally the mathematics area should be a free-standing worktop where children experiment and display their results. In addition it is necessary to have wall space for displaying charts, flashcards and the results of the children's work. The worktop space could be a cupboard or shelving which can then be used to store equipment not in use. Mathematical displays and apparatus should be changed to suit the strand being worked on if they are to be seen to be effective and genuine aids.

### Effective use of equipment

Children who are actively involved in a structured task will be more likely to exhibit positive classroom behaviour, and the teacher will be free to work with another child or group of children. It is important that the children share responsibility for the appropriate use and storage of the apparatus, as this will develop their independence.

Charts showing labelled equipment and the terminology in use should be visible to those working in the area. These give the child the freedom and independence to work on tasks uninterrupted. If possible there could be a recording area nearby, or children could use clipboards for on-the-spot recording. Colour-coded or number-coded pockets of worksheets could also be provided so that the children can work independently.

## Mathematics in the senior classes

Mathematical exploration in the senior classes should continue the use of manipulatives. It is envisaged that there will be an even greater emphasis on problem-solving and the use of examples from real-life situations, for example using information from newspapers or local shops. Calculators may be used for handling larger numbers but children should be encouraged to make decisions about when they need to use them and to be confident in their use. This is where their developing estimation skills will be important.

Where computers are available they will be of great assistance in cross-curricular projects for data representation and interpretation and as a basis for drawing conclusions based on data collected.

Work on simple percentages and their relationship to fractions and decimals can be related to examples of their practical use in the environment and in advertising. Numbers should be kept small and simple to encourage consolidation of concepts.

Work on shape and space should, where possible, involve handling and manipulating shapes.

## Individual difference

The content of the mathematics curriculum is sequential and dependent on knowledge gained at each level. This needs careful planning, as children acquire the requisite skills in different ways and at their own individual pace. In planning a sequence of lessons on a topic the teacher must first assess the readiness levels of the children. This can be done by giving a pre-test or when doing revision. The information gained can be used to group the children where necessary. Periods of direct whole-class instruction can follow, with the children contributing to blackboard or overhead projector work at their own level. Emphasis should be placed on the quality of the contributions, for example how they arrived at a conclusion, how some children found a different method, summarising what has been done, or identifying strategies that might be useful in approaching a task.

Games can be very useful in mathematics. Card and dice games can reinforce number recognition and help in the development of strategies. They also encourage co-operation and turn-taking. These activities should be structured by the teacher, and he/she can discuss with the children why they chose to play the game a certain way. Older children can design their own board games. They will be quick to notice what happens

when they put in too many penalties, for example too many snakes in a snakes and ladders type game. The game ceases to be fun if there is little chance of winning.

Children often fail at mathematics because they have missed out on an earlier learning experience, for example one-to-one correspondence, seriation or conservation. Language or reading deficiencies can inhibit the child in approaching written problems. This can often be overcome by presenting tasks concretely, pictorially, diagrammatically or with pictures to support the words. Poor sense of direction, time and spatial relationships can also interfere with the learning of mathematics, and memory deficits mean that the child cannot easily recall number facts.

In planning sequences of instruction the teacher must consider these factors and encourage the development of alternative strategies. The establishment of personal benchmarks in measuring can be of great help, for example *the width of my little finger is about one centimetre, if I stretch out my arm it is about a metre from the tip of my finger to my neck*. Labelled reference points in the classroom can also assist the child in estimating heights and widths, for example *the bookshelf is one-and-a-half metres high and I am nearly as tall as it, four carpet tiles make a square metre*.

All children gain from using strategies for number facts. They can learn the 'easy' number facts first ( $\times 1$ ,  $\times 2$ ,  $\times 5$ ,  $\times 10$ ) and use these to build up the others using doubles, near-doubles and patterns of odd and even. These strategies are of particular help to children with memory problems.

It is also important to consider the child who may be particularly good at mathematics. He/she can be given more difficult or taxing problems to solve rather than prematurely pushing him/her forward. Problems with two or three steps or open-ended problems are more difficult and provide a challenge. Once a concept is well understood it is better to use it in problem-solving activities than to overuse rote computational exercises. Sequences of graded work-cards allow children to work at their own pace and to undertake extension activities.

A balanced mathematics programme will cover concepts, skills and problem-solving and should consider the child's strengths and weaknesses. Computer technology and calculators can be used effectively both in remediation work and in extension activities for the more able child.

## Assessing children's work in mathematics

Assessment is an integral part of the teaching and learning process. Teachers use assessment techniques every day. They make decisions about what to teach and how to teach it based on their observation of the children and the feedback they receive from work the children are doing. Reporting to parents is usually based on both the results of tests and the teacher's assessment of the child's approach to the subject.



# Approaches and methodologies

# Approaches and methodologies

## Changes in methodology

- more emphasis on guided discussion
- hands-on approach from infants to sixth class

## Discussion skills

- turn-taking
- active listening
- positive response to the opinions of others
- confidence in putting forward an opinion
- ability to explain clearly their point of view

## Teaching approaches

Teachers will notice that the changes in this curriculum cover both content and methodology. New topics have been introduced and other topics have been omitted, repositioned or given a new emphasis in the curriculum. It is hoped that these new emphases will lead to an enhancement of the child's mathematical education and to a heightened pleasure and interest in the subject.

## Guided discussion and discussion skills

This curriculum places great emphasis on child-child and child-teacher discussion. In this way the child clarifies his/her thinking and gains self-confidence and self-esteem. Discussion, rather than just questioning, should be the basis of the interactions between teacher and child. This strategy encourages the development of skills and is also the arena for developing mathematical language.

Children must be trained in discussion skills before they can effectively use them in a group. Skills such as turn-taking, responding positively to the opinion of others and having the confidence to put forward an opinion of their own are essential skills that transfer both throughout the curriculum and into real life. Children must be secure in the knowledge that others will listen to their opinion and treat it with respect.

## Using a hands-on approach

A hands-on approach is essential if children are to understand mathematical concepts. This approach is important right through to sixth class and will require access to a considerable amount of equipment. Working with equipment can be done individually, in pairs or in groups, and the allocation of the equipment can be organised on a class or school basis.

## Mathematical language

When children use mathematical language it is important that they use it accurately. Understanding mathematical language leads to the correct interpretation of mathematical symbols and accurate reading of algorithms or word problems. This helps the child to choose the correct operation for the task.

Discussing and interpreting symbols in the environment is a good starting-point for introducing mathematical symbols as well as being a learning exercise in itself. Signs often have words on them, while symbols are usually pictorial representations of a statement, for example the no smoking sign, road signs, poison and cleaning instructions on clothes. These are internationally recognised symbols and indicate to the child that these symbols carry a meaning with them, as do mathematical symbols.



It is helpful when teaching to have a common approach to the terms used and the proper use of symbol names. Introducing mathematical symbols and numerals is the last step in the learning process. In teaching place value it is better to use units than ones, as it can be confusing for the child to describe 21 as 'two tens and one one'. Work on place value could include collecting the house numbers of the children in the class and classifying them as being numbers with one digit, two digits or three digits.

The same care and attention should be given to the formation of numbers as is given to the formation of letters. Children should practise forming and writing numerals in sand, feeling the numeral on sand-paper or carpet, and tracing it. They should be given clues and guidance on where to start. This exercise can be reinforced by means of charts. The teacher can observe children who have difficulties with, for example frequent reversal of numbers, poor spatial awareness or poor manual control.

It is a particularly good idea when teaching and assessing the child's concept of place value to present the algorithm horizontally. The pupil then has to find the value of each digit before writing the algorithm vertically. It is important that children read algorithms from left to right. This is similar to left-right orientation exercises in reading and writing.

When children see  $259 - 156$  they should be encouraged to 'read' it as *259 minus, subtract or take away 156*. This makes it clear to the child that the smaller number is the subtrahend. Children often misinterpret division statements, for example  $410 \div 7$  could be read by the child as *410 into 7*. It should be read as *410 divided by or shared between 7*.



# Estimation strategies for number

## Estimation procedure

- estimate first
- write down your estimate
- solve the problem
- compare your estimate with the actual result

Throughout all the strands of this curriculum emphasis has been placed on the development of estimation strategies. Estimation is the process of taking an existing problem and changing it into a new form that is easier to compute mentally and gives an approximate answer. This skill is essential for real-life mathematics, for example shopping or measuring time and distances.

From the very early days at school children need to be encouraged to estimate. The young child finds it difficult to differentiate between 'estimate' and 'answer'. The teacher will have to lead the work by encouraging them to make a *sensible* 'guess', to test their guess and revise it where needed. Estimation is a help towards finding a solution but need not in itself be the solution. This is important in measurement activities, where the children can be encouraged to compare objects as being '*a good bit longer than ...*' or '*only a little heavier than ...*'

Children must also be taught to investigate the reasonableness of their results. They can be encouraged to develop their own ways of deciding when an answer is reasonable. They can be presented with a problem and several solutions, from which they select the most reasonable solution.

Estimation is also necessary for work with calculators so that the child can evaluate the validity of the result given by the machine. Quick recall of number facts and a strong number sense are important for efficient estimation. There are many different approaches to computational estimation, and good estimators use a variety of strategies.

## Front-end strategy

This strategy has its strongest application in addition. The left-most digits (front-end) are the most significant in forming an initial estimate and can be used on their own in the earlier stages to establish a rough estimate:

$$\begin{array}{r} \text{£} \quad \text{p} \\ 1 \ . \ 54 \\ 6 \ . \ 35 \\ 0 \ . \ 99 \\ \underline{2 \ . \ 51+} \end{array}$$

front-end process:

add the front-end amounts:

$$\text{£}1 + \text{£}6 + \text{£}2 = \text{£}9$$

adjust by grouping the pennies to form pounds

$$54\text{p} + 35\text{p} \text{ makes } \text{£}1 \text{ approx.}$$

$$99\text{p} \text{ is nearly } \text{£}1$$

$$51\text{p} \text{ is nearly } 50\text{p}$$

pennies estimate: £2.50

overall estimate is £11.50 (£9 + £2.50).

This strategy can be introduced by using money initially but works equally well with whole numbers, fractions and decimals. The adjustment stage can be introduced gradually as the children become familiar with the concept of 'nearly £1' or 'nearly 50p'.

It can also be accomplished with multiplication for example,

$$369 \times 6$$

$$300 \times 6 = 1800$$

$$70 \times 6 = 420$$

Estimate is 2220

### Clustering strategy

This is best suited to groups of numbers that 'cluster' around a common value, for example

#### Numbers of people who came to our concert

Monday 425

Tuesday 506

Wednesday 498

Thursday 468

Friday 600

The average attendance was about 500 per night.

$$500 \times 5 \text{ nights} = 2500.$$

### Rounding strategy

Numbers can be rounded in many different ways. The choice of rounding process will produce different but reasonable results, and this can be refined according to the child's ability to compute mentally. It is necessary to give children plenty of mental practice with this method and demonstrate how it can be refined by choosing closer rounding factors. Using this strategy can generate plenty of discussion about why one child's answer is different from that of another.

$37 \times 59$ : in this case it would be best to round both numbers *up*:

$$40 \times 60 = 2400$$

$51 \times 22$ : here we would round both numbers *down* to 50 and 20:

$$50 \times 20 = 1000$$

$24 \times 65$ : they are both close to the middle so you can try rounding one *down* (20) and one *up* (70):

$$20 \times 70 = 1400$$

Rounding can be used with the four operations but is very useful in division. In division it is often better to round *up*:

$419 \div 65$  could be rounded to  
 $420 \div 70$ .

### Special numbers strategy

This strategy looks for numbers that make patterns, for example tens or hundreds

(a) 3	(b) 37
5	54
7	71
4	42
<u>6+</u>	<u>69+</u>

(a) 3 and 7 are ten, 6 and 4 are ten, that's 20; add the 5, this totals 25

(b) older children could group the tens using a mixture of rounding and compatibility, for example 37 and 42 is about 80 ...

Estimation skills are essential throughout the strands and at all class levels. These skills can be used in *Measures* in conjunction with using a known unit, for example nearly a metre, less than a litre, about half a kilogram and in fractions and decimals: close to 0, close to  $\frac{1}{2}$ , close to 1.

# Problem-solving

## Mathematical problems include:

- word problems
- practical tasks
- open-ended investigation
- puzzles
- games
- projects
- mathematical trails

## Clue sheet—solving a problem

- examine the problem:
  - what does it tell me?
  - what does it ask me to do?
  - how will I do it?
  - have I all the information I need?
- solve the problem
- have I done what I was asked to do?

The child's attempts to solve a problem require him/her to call on many skills. Problems in mathematics have often been seen as textbook examples at the end of a section on a particular topic. Problems in life are rarely that simple, and there is often more than one way to find a solution.

Problem-solving experiences should develop the ability to plan, take risks, learn from trial and error, check and evaluate solutions and think logically. Discussion and acceptance of the points of view of others are central to the development of problem-solving strategies.

Problems can be classified in many ways. They can be presented concretely, diagrammatically or in written form. They can be open or closed. They can relate to one particular content area or include elements from more than one strand.

A written problem may be difficult to solve because of readability or because it has multiple steps to the solution procedure. Large and awkward numbers often frighten a child away from attempting a problem, and if the information is not presented in the order in which it is to be used some children just give up without trying. If children are taught to analyse the problem carefully and extract the relevant information they can often find that it is much easier to solve than it appeared at first.

Children need to develop problem-solving skills in general and to be confident in their own ability to attempt a solution.

- children should be taught a variety of strategies and to experiment with applying the same strategy to different problems and different strategies to the same problem. These strategies will vary according to the age of the child
- the teacher will need to structure the problems given to the children so that they experience success
- re-reading of a problem by the child should be encouraged
- co-operative group work and class discussion of the results of a problem-solving exercise encourage the children to respect the ideas of others, to try different approaches themselves, to offer alternative solutions and try them out on the blackboard
- giving the children problems with irrelevant information or with no solution possible because of missing information encourages them to analyse what it is they are being asked to do; for example,

*Jim has red hair. His sister gave him 50p. He bought a bar for 20p and a bag of crisps for 30p. Do you think Jim's sister was a kind girl?*

Although the numbers appear to add up, the child was not asked to operate on them. This type of exercise can be done orally and encourages a more critical approach to a problem

- children can invent problems for others to solve, and discuss the results.

### Problem-solving strategies

Problem-solving strategies must be varied and the children given ample opportunity to try them out concretely, orally or in a written task. Many children fail at mathematics because their mathematical vocabulary is insufficient to cope with the terminology of problems. Development of the necessary vocabulary in a consistent manner throughout the classes must be stressed. Some strategies that can be taught to children are:

- constructing a model
- drawing a diagram to illustrate a problem
- making a chart or table of the information
- looking for patterns in a problem
- making a guess and testing it out
- breaking the problem down and solving each part
- writing a number sentence for the problem
- using appropriate equipment to solve the problem, for example balance, measuring instrument, calculator, blocks
- solving a simpler version of the problem, for example using smaller numbers.

# Some activities with odd and even numbers

When children have a conceptual understanding of multiplication as repeated addition or grouping, the multiplication sign can be introduced as meaning 'sets of'.

The children can be given a worksheet with the multiplication facts arranged in table form and asked to fill in as many of the facts as they can from memory. Reference should be made to previous work done by the children, for example patterns on the hundred square. After filling in the multiplication table from memory the children can discuss with the teacher what strategies they used to help them fill in the products they did not know. Initially the most common strategy for working out an unknown fact is counting on from a known fact: for example,  $3 \times 6 = 18$ , so  $4 \times 6$  is  $18 +$  another 6. Some children will use skip-counting from memory or count on their fingers.

In exploring multiplication patterns children will notice that odd and even patterns appear, and this can provide scope for interesting investigations; for example will the final digit in the  $\times 2$  facts be odd or even? What about  $\times 4$ ? or  $\times 3$ ?

These investigations can lead to further work on odd and even. Children can work in pairs with one child being 'odd' and one child 'even'. They each throw a die and get a point each time an odd or even number is thrown. They will soon notice that there is an equal chance of either of them winning as there are three even and three odd numbers on the die. This can be extended by using two dice. The initial response may be that the result will be the same (equal chance of winning). The children will soon realise that you must add the totals of each die, as it is the total that scores the point. After the experimentation and guesswork it is interesting to record and discuss all the possible combinations of the two dice throws.

## Odd totals

$1 + 2 =$	3
$1 + 4$ or $2 + 3 =$	5
$1 + 6$ or $2 + 5$ or $3 + 4 =$	7
$3 + 6$ or $4 + 5 =$	9
$6 + 5 =$	11
Number of possibilities:	9

## Even totals

$1 + 1 =$	2
$1 + 3$ or $2 + 2 =$	4
$1 + 5$ or $2 + 4$ or $3 + 3 =$	6
$2 + 6$ or $3 + 5$ or $4 + 4 =$	8
$4 + 6$ or $5 + 5 =$	10
$6 + 6 =$	12
Number of possibilities:	12.

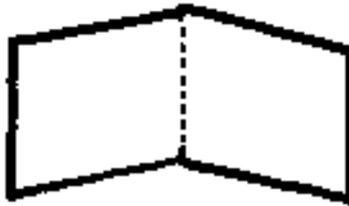
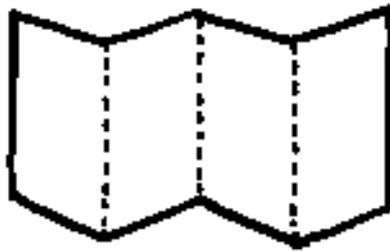
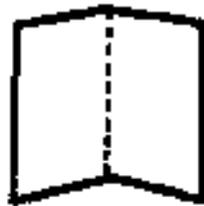
# Paper-folding and fractions

Paper-folding can be used effectively in the development of fraction concepts.

The teacher and the children fold a sheet of paper in two and discuss the equality of the parts. The concept of half can then be introduced and the children can colour half of the piece of paper. The same activity can be done with quarters by folding the paper a second time.

*Is there more than one way to fold a piece of paper in half? in quarters?*

The children will have to consider the size of the unit.



## Comparison of fractions

*Which is bigger, a half or a quarter?*

*By how much?*

Children show how much bigger the half is and discuss how they found out.

*Which is bigger, a half or three quarters?*

*By how much? Which is smaller?*

## Equivalence of fractions

*Which is bigger, a half or two quarters?*

This can also be done by re-folding and discussing eighths.

*Which is bigger,  $\frac{3}{4}$  or  $\frac{5}{8}$ ?*

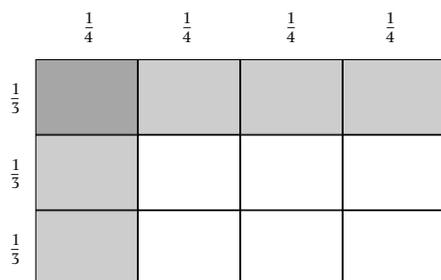
*Which is smaller,  $\frac{1}{2}$  or  $\frac{3}{8}$ ?*

*Which is bigger,  $\frac{2}{4}$  or  $\frac{4}{8}$ ?*

Similar work can be done with thirds and sixths. Folding becomes difficult for fifths and tenths so it is best to develop the concept with fraction families that are easy to fold. However, similar work can be done on squared paper.

## Addition and subtraction within fraction families

Add  $\frac{3}{4}$  and  $\frac{1}{8}$ ; add  $\frac{1}{3}$  and  $\frac{1}{6}$ .



### Cross-folding

To compare fractions from different families it is necessary to use cross-folding.

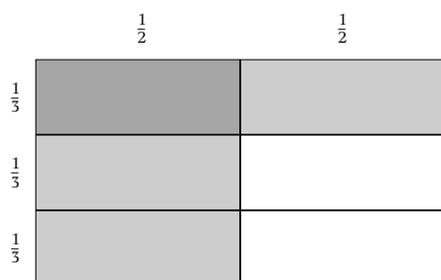
Fold *one* sheet to illustrate quarters and thirds. Fold one direction for quarters, the other direction for thirds.

*Which is bigger, a quarter or a third? Why?*

*How many equal parts is the unit divided into?*

*How many in one third? One quarter?*

*Which is bigger, two thirds or three quarters? one third or two quarters?*



### Addition

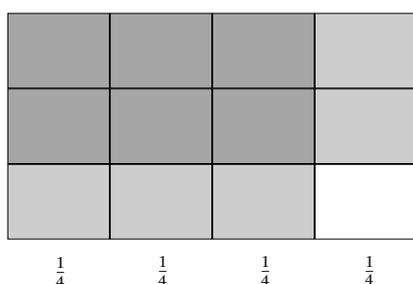
Add a half and a third. Fold the sheet into two halves in one direction, and three thirds in the other direction.

*How many equal parts?*

*How many equal parts in one half?*

*How many equal parts in one third?*

*How many in a half and a third?*



### Subtraction

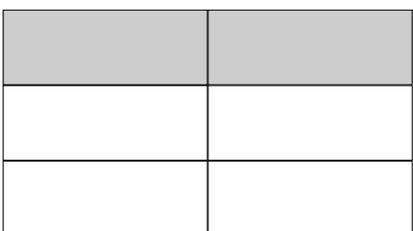
Subtract two thirds from three quarters by folding in two directions.

*How many equal parts?*

*How many equal parts in three quarters?*

*How many equal parts in two thirds?*

*How many left when two thirds are removed from three quarters?*



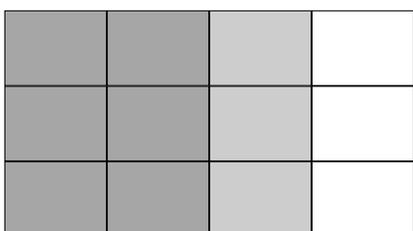
### Multiplication

Fold a sheet of paper into halves in one direction and into thirds in the other direction.

*How many equal parts?*

*How many equal parts in one third?*

*How many in half of one third?*



Use the same method to find: a) two thirds of three quarters  
b) three quarters of two thirds

*Are they the same?*

Use the same method to find three fifths of a third.

# Early mathematical activities

## Activity

Matching equivalent and non-equivalent sets using one-to-one correspondence

## Class

Junior infants—first term

*children will engage in these activities over a period of weeks.*

The aim of this activity is to establish relations through matching. One-to-one correspondence is the basis for counting. The approach used is discovery learning, and the skills covered include recording, reporting, predicting, sorting, classifying, investigating, planning and co-operating. The children do the activity, discuss the results with each other and with the teacher and then record their results.

## Equipment

pairs of identical or related objects, charts showing labelled objects

packs of cards, for example Happy Families, Snap

dominoes

overhead projector

television programmes that portray matching activities

## Recording

pictorial recording, non-permanent recording using string, cards or arrows and oral recording. No symbols or number names are used at this stage.

## Preparation

discuss language of lesson informally

discuss recording, e.g. the arrow

identify equipment

## Language

match, goes with, belongs to, each,

more than, less than, enough, not enough

## Lesson content

Children match 1 cup ↔ 1 saucer

Children match 3 flowers ↔ 3 flowers

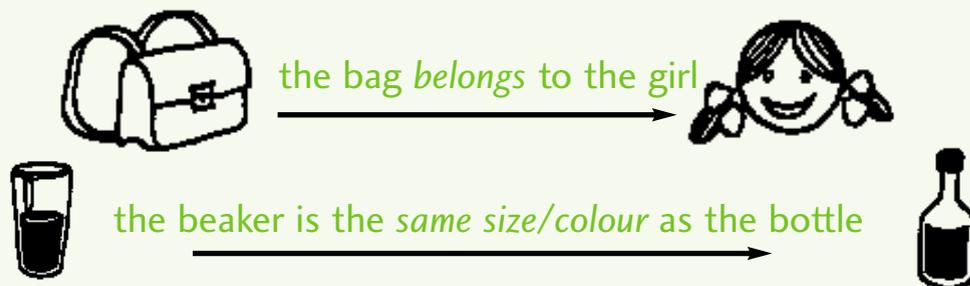
Children match 5 buckets ↔ 5 chairs

Children match 4 flowers ↔ 2 flowers

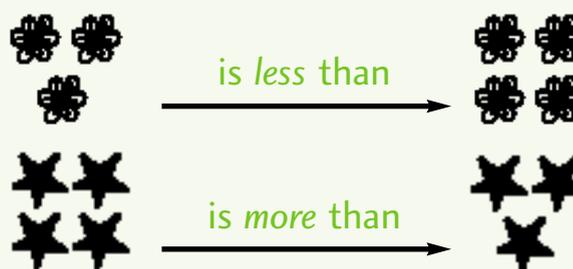
Children match 3 children ↔ 1 pencil

Group of children and chairs. *Is there a chair for each child?*

Establish relations and discuss.



Discover inequality and discuss.



- reinforce these concepts using matching picture cards, dominoes and peg boards



- make a necklace using one bead of each colour
- cards with pictures of objects, children find the same number of counters
- match cards with the same number of objects
- record orally and pictorially with child-child and teacher-child discussion
- children set up similar challenges for each other
- teacher chooses a child to drop a bead in a box for each child as they leave the room, another child takes out a bead as each child returns.

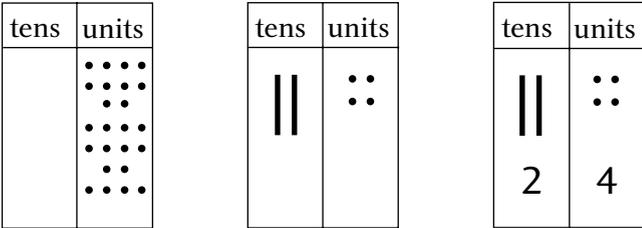
*what does it mean when box is empty?*

*what does it mean when beads are left?*

*what does it mean when box is empty before all are seated?*

# Place value notation boards

Place value notation boards may be used to enhance the child's conceptual understanding of the value of digits. In first class children are required to group and count in tens and units and express groups of counters as units or tens and units. Using the 'board' is a very effective way of engaging in this activity. Numeral cards may be used in conjunction with these boards.

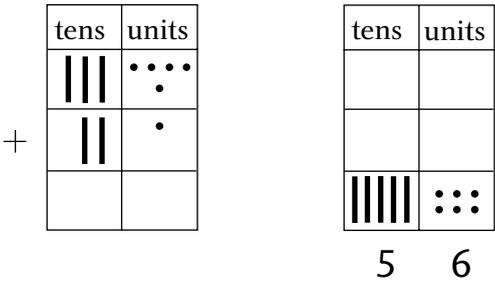


Later, when the child is required to add tens and units, these boards are again very useful.

## Example A: Addition (no regrouping), 35 + 21

Step 1  
Using base ten materials represent the numbers to be added

Step 2  
The child combines/ adds the units and then the tens



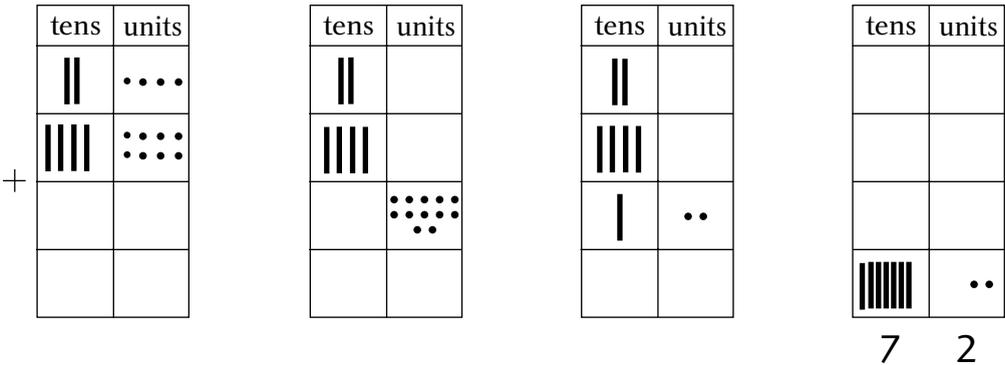
## Example B: Addition (involving regrouping), 24 + 48

Step 1  
Represent numbers to be added

Step 2  
Add/combine units

Step 3  
Regroup units as tens and units

Step 4  
Add/combine tens



## Place value notation boards

When subtracting, the boards are used in a similar way. Only the greater, however, is represented by base ten materials on the board. Numeral cards represent the number of units and tens to be subtracted.

### Example A: Subtraction (no renaming necessary), $58 - 23$

**Step 1**  
The child represents 58 using base ten materials and 23 using numeral cards

tens	units
	::::
2	3

**Step 2**  
The child takes away three units, placing them on the '3' numeral card and two tens, placing them on the '2' numeral card.

tens	units
	:::
2	::3

**Step 3**  
The remaining units and tens, what's left, may be positioned in the 'answer slot' at this stage.

tens	units
2	::3
	:::
3	5

### Example B: Subtraction involving renaming, $64 - 38$

**Step 1**  
The child represents 64 using base ten materials and 38 using numeral cards.

tens	units
	::::
3	8

**Step 2**  
The child cannot take away 8 units. He/she exchanges one of the 6 tens for ten units. He/she now has 5 tens and 14 units on his/her board

tens	units
	:::::
3	8

**Step 3**  
He/she takes 8 units from the 14, placing them on the '8' numeral card and 3 tens from the 5, placing them on the '3' numeral card

tens	units
	:::::
3	::::8

**Step 4**  
The remaining tens and units, what's left, may be positioned in the answer slot.

tens	units
3	::::8
	:::
2	6

Using place value notation boards for subtraction in this way will enable the child to recognise the relationship between addition and subtraction.

The boards depicted on the previous page show only tens and units, but in third class the boards would include a column for hundreds, and in fourth class place value notation boards would have four columns, thousands, hundreds, tens and units.

Simple games can be played by groups or pairs of children, using dice, base ten materials and notation boards, during which children attempt, for example to reach 'target numbers.' Such games could involve addition or subtraction.

### Addition and subtraction transition boards

Transition boards are simple devices to aid childrens' conceptual understanding of addition and subtraction. Transition boards, foster the linking level of addition and subtraction in base ten.

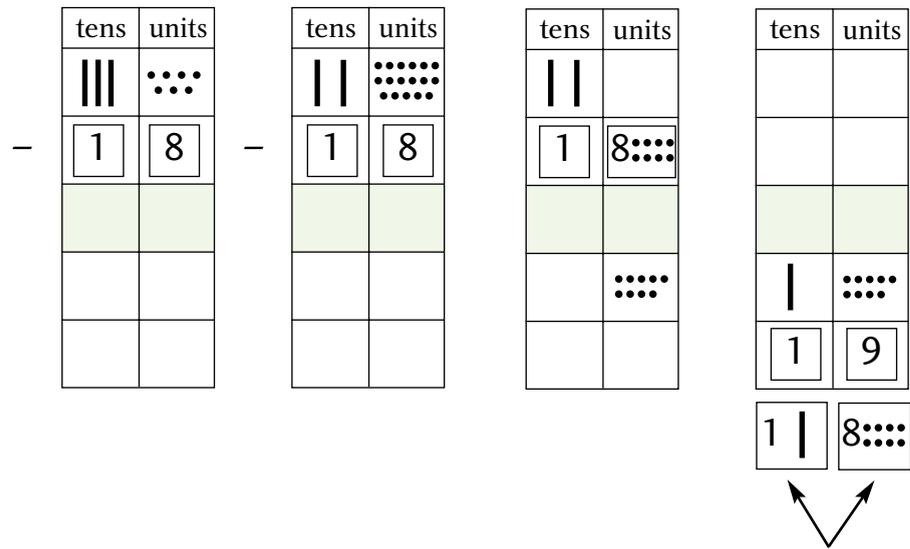
The general strategy for teaching is to lead the children through the concrete, linking and abstract levels as follows:

- they work only with the manipulatives on the transition board as the teacher orally states the problem while modelling on the overhead projector
- they write the numbers on the chalk board and use the manipulative pieces as they work with the materials on the transition board
- they represent the numbers with pictorial models of the manipulatives on a small paper model of the transition board
- they write the numbers in the spaces on a small paper model of the transition board
- they write the numbers on their paper.

### Sequential use of addition board, $34 + 27$

	tens	units		tens	units	tens	units	tens	units
		::							
		:::							
+					●●●●				
							.		
									.
								6	1

### Sequential use of subtraction board, $37 - 18$



The above subtraction tickets show the amount taken away from 37, i.e. 18 (one ten and eight units).

# Integration, linkage and cross-strand planning

**Integration:**  
a cross-curricular approach

**Linkage:**  
integration within  
mathematics

**Cross-strand planning:**  
encourages a balanced  
approach when teaching the  
strands

The **integration** of mathematics with other subjects is an important factor in broadening the child's education. Elements of *Measures* can be applied when measuring the length of jumps taken or distances run in physical education activities. Scale is used in map reading and timing in orienteering exercises. *Shape and space* activities are particularly useful in both physical education and geography as children develop a sense of location and space. It is interesting to look at tessellation in relation to mosaics or to examine the geometric elements of a Greek temple. Investigation of sundials, water clocks, roman numerals and the history of old coins can enrich a child's understanding of time and money.

In science, working scientifically and using problem-solving approaches naturally encompass mathematics. Selecting appropriate methods of recording, analysing results of investigations and identifying variables in designing a fair test offer opportunities for using mathematics in interesting and real situations. Children do not naturally make connections between work in one curriculum area and that in another, so it is important to help them to understand that mathematics is useful in their work in other areas: for example sorting animals with wings or shells is just as valid an activity as sorting coloured buttons.

Integration adds to the child's enjoyment of mathematics, gives him/her added interest in the subject and encourages transfer of learning.

**Linkage** is integration within a subject area. It is not necessary to complete the *Number* strand before proceeding to, for example, *Measures* or *Shape and space*. As skills in number are established they can be applied within the content of the other strands. This linkage within the mathematical programme can be likened to the building of a jigsaw. All pieces are necessary and are part of the entire picture. Textbooks should be used in a way that supports that strategy. Strands should not be taught in isolation. Strand content from one area supports and forms the basis for learning in another strand. Linkage provides balance in the teaching of all the strands.

Linking aspects of the mathematics curriculum with each other involves **cross-strand planning**. It is suggested that elements of the five strands be covered on a rotational basis. The sample timetable on page 59 is an example of how this can be achieved. Not all the strands are included in this plan. It is envisaged that those remaining will be covered when the teacher is organising the next plan. What is important is that strands are introduced gradually and complement one another. Teachers will develop their own timetables, ensuring that all strands are adequately represented.

# Mathematical trails

Trails in local history and in geography have become an accepted part of the curriculum. It is also possible to devise mathematical trails. A trail is an activity-based assignment that can take place within the school building, in the school yard or in the locality.

Here are some examples of types of trails that can be devised:

- a trail based on a mathematical strand: for example, *Measures: Weight*, includes linkage with *Number, Shape and space, Data*
- a topic-based trail using linkage within mathematics and integration with other subjects, for example a trip to a shopping centre or maths in our town.

## Why use mathematical trails?

- they add a sense of fun and adventure to mathematical topics
- they offer an alternative to formal, written work and are challenging to the children
- trails can be structured to include work suitable for all levels of ability. Children who have previously experienced difficulties in mathematics can be stimulated by this approach, and the abilities of more able children can be challenged
- this type of integrated approach shows the child how mathematics is useful throughout the curriculum

- it is a socially interactive activity, which allows every child to undertake responsibility. Observers and recorders play an equal role in the activity
- it offers an opportunity to revise aspects of many of the strands and to assess progress in an integrated manner.

## Organising a mathematical trail

The teacher plays a vital role in organising a trail. The topic and venue will have to be selected and decisions made about how the task will be presented.

- will presentation of the assignment to the children be written or pictorial?
- will the bulk of the recording be done while on the trail or in the classroom?
- a decision must be made whether the trail is more suitable for the whole class or a section of the class
- parents, classroom assistants or senior children can be enlisted as helpers
- equipment must be organised in advance
- the time allocation must be clearly defined
- the aim and purpose of the exercise should be explained to the children and follow-up activities organised.

### Pre-trail work

The proposed route, rules of behaviour and co-operation should be discussed with the children. They must also consider the country code, road safety and respect for people and property. The children need to understand clearly the purpose of the exercise. If the trail is to include a shopping centre or public venue it is often advisable to contact someone in authority in advance to avoid possible difficulties.

The teacher must set a starting and finishing time and set a time limit for the completion of each stage. He/she must give clear instructions on how to record results and ensure that all children are actively involved.

### Follow-up work

- complete the recording and integration work
- display work where possible and discuss the results
- discuss with the children what they have achieved, what they enjoyed most and how they could improve a future trail.

### Extension work

Children can devise their own trail. It could be based inside or outside the school. A trail on their own street could be devised and completed as homework. Senior children could design a trail for junior classes or a tourist trail around their own locality.



# Exemplar 1

## Trail: Delivery of the post

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### A trail based on the work of a postperson.

#### Type of trail:

Topic-based, including mathematical linkage and cross-curricular integration

#### Venue:

Outside school

#### Class:

Middle classes, children in groups

#### Mathematical aspects of the trail

*Number, Shape and space, Measures (Length, Area, Weight, Time and Money), Data*

#### Integration

English, Gaeilge, SESE, Arts education

#### Aims

- observe and encounter mathematics while on the trail
- apply mathematical knowledge while on the trail
- enjoy the mathematical experience
- co-operate and interact as teams
- revise mathematics completed so far in an interesting and stimulating way
- transfer mathematical concepts learned in the classroom to practical experience.

#### Materials

worksheets, metre-stick, string, measuring-tape, trundle-wheel, watch, pencil, paper, eraser, parer, clipboard, holdall, route map, camera (optional)

#### Pre-trail preparation

- teacher investigates route, draws simple map and identifies stopping-places
- class collects letters and cards; make parcels to have a post-bag
- collect information about postal services.



### Implementation options

Each group can complete all the worksheets or just one and report back on return to the classroom. Each worksheet can be based on one particular strand or can have mixed questions.

### Extension work

Extension work could include an investigation into the quickest and most efficient way to deliver letters. A visit to the sorting office or a visit by a postperson to the class could be arranged. Collect data about the post delivered to the school in a week.

This sample mathematical trail can be adapted to suit all classes in an urban or rural school, for example a country road trail, a shopping centre trail, the farmer's day, the shopkeeper's day or the shopper's day.

### Delivery of the post

#### Suggestions for developing a worksheet: Number

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1. Write the number of any five houses on the left-hand side of the road. Mark the odd and even numbers. Multiply the first number by the last number.
2. How many houses on a given stretch of street? Letters are delivered to half of the houses and parcels to a quarter of the houses. How many gates did the postperson open?
3. Two letters were delivered to each house on the right-hand side of the road each day. How many letters were delivered in a week?
4. Count the windows in one-storey and two-storey houses on both sides of the street. Which type of house had most windows?
5. What is the telephone number in the telephone box? Subtract the smallest digit from the largest digit. What is the largest/smallest three-digit number you can make from the digits? Subtract the smaller number from the larger one.
6. Write down the registration number of a car. How old is the car? Divide the registration number by the last digit.
7. If every house on a particular road received five letters on Monday, six on Tuesday, eight on Wednesday, one on Thursday and four on Friday, how many letters were delivered altogether?

## Delivery of the post

### Suggestions for developing a worksheet: Shape and space

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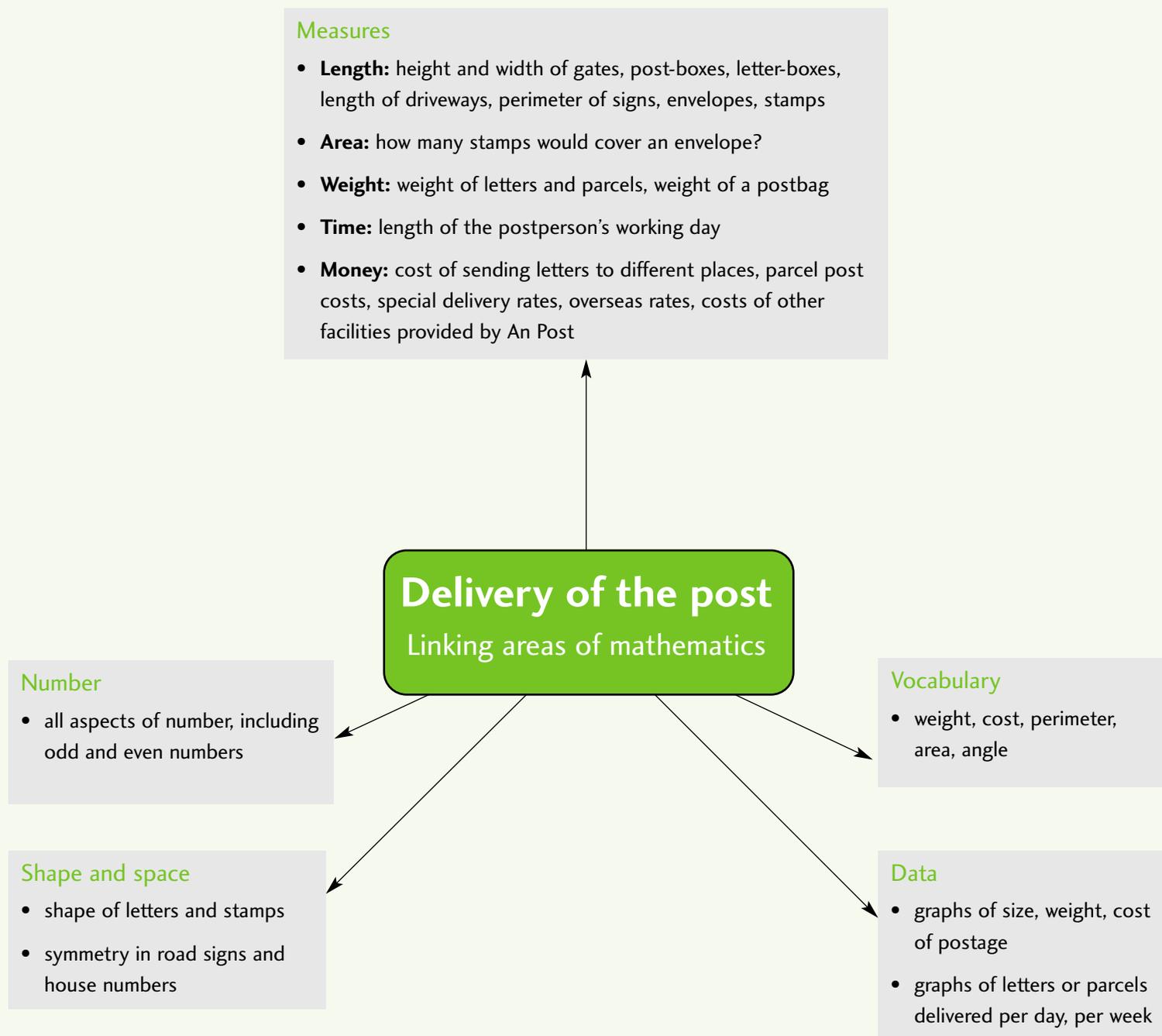
1. Draw the sign on telegraph pole number 6 on your map. What does it mean?
2. Look at house number x. How many right angles do you see? Draw the house.
3. Draw four gates that you pass on a particular stretch of road. Count the vertical and horizontal bars: how many of each? Find a gate that has the same number of parallel and perpendicular bars.
4. What is the Irish name for street or estate x? What shape is the sign? Draw the sign and write the name.
5. Look at the 'no entry' sign; what shape is it? Draw it. Is it symmetrical?
6. Draw any cylindrical shapes you see and label them, for example a post-box, a telegraph pole.
7. Find the following shapes on the trail: triangle, parallelogram, pentagon, octagon. Draw and label.

Further worksheets for this trail can be devised for *Measures* (Length, Area, Weight, Money and Time) and for *Data*.

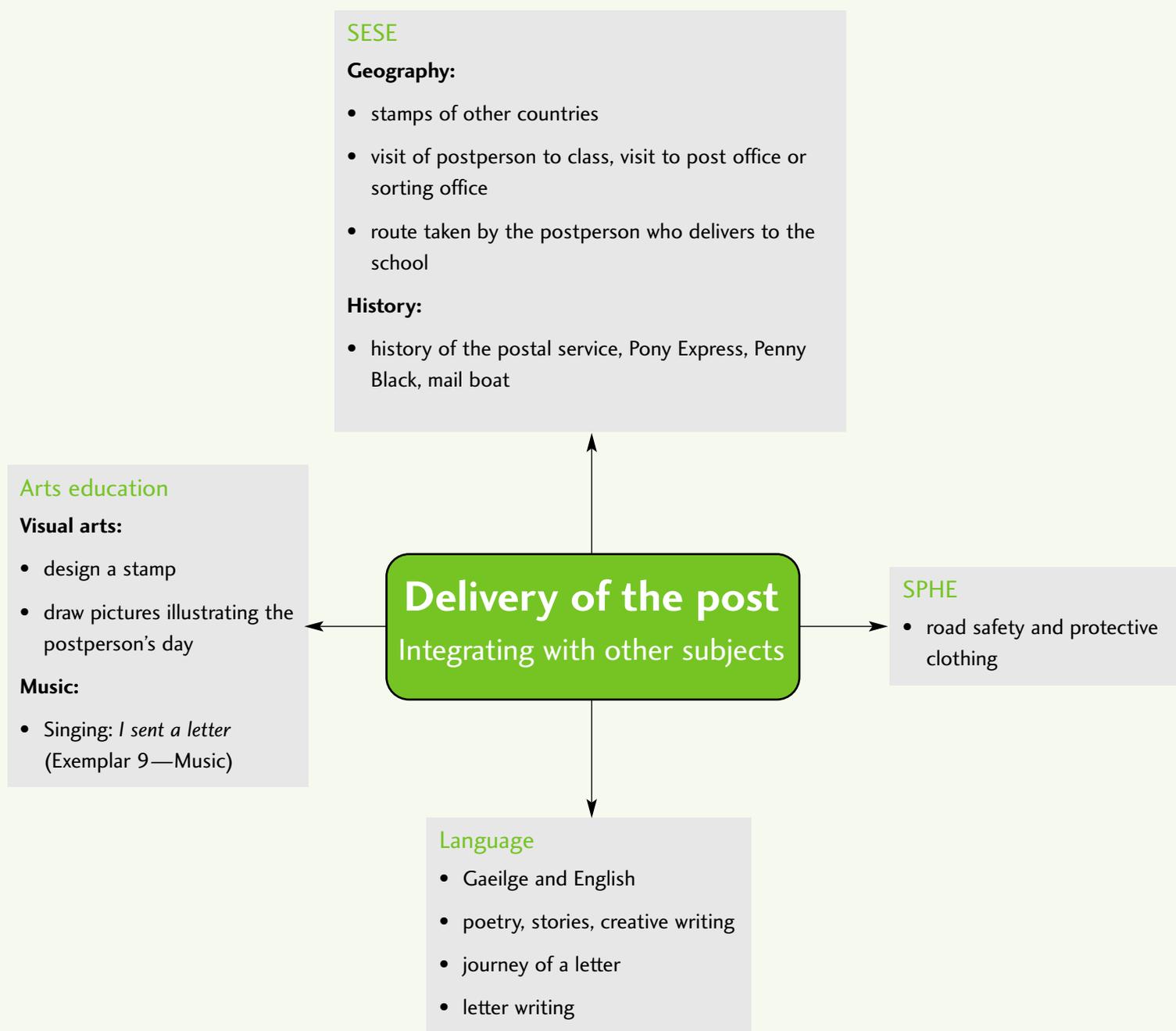
### Examples

1. Estimate and measure the height and circumference of a post-box.
2. Examine the speed limit sign. What shape is it? How long would it take you to travel 250km at that speed?
3. Look at the post-box. Note the times of collection of post. How many hours or minutes between collections?
4. Collect information about house types on your route. Draw a pictogram showing the different house types.
5. Make an envelope that fits criteria set by An Post.

## Suggestions for linkage: Delivery of the post



## Suggestions for integration: Delivery of the post



# Exemplar 2

Length: mathematical topic illustrating linkage

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**Class:** Second class—children in groups of 5 and 6

**Linkage:** *Number, Algebra, Shape and space, Measures, Data*

**Materials:** worksheets, string, rulers, straws, cubes, lollipop sticks, unmarked colour-coded measuring sticks, metre stick, tape-measure, trundle-wheel, coins, stop-watch

**Implementation:** each group can complete all six worksheets or just one worksheet  
one group can work at a time while the remainder of the class work on other activities

## Suggestions for worksheet 1

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1. Estimate the length of two given ribbons. Record your estimate. Choose a measuring tool, measure. Find the total length.
2. Walk to spot X. Turn right, walk to spot Y. Estimate how far you walked. Write down your estimate. Choose a measuring tool and measure the length of the walk.
3. Choose a piece of string. Estimate and record the length. Find the length. Find the cost of the string at 5p per centimetre.
4. Choose the tallest person in the class. Measure his/her height with string. Measure your own height with string. Cut a piece of string to show the difference in your heights. Discuss.

## Suggestions for worksheet 2

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1. Draw 3 straight and 3 curved lines on a large sheet of paper. Estimate and record the length of each of these lines. Choose a measuring tool and measure the lines. Record your results.
2. Fold a metre length of paper in two. Fold it in half again. Cut the four pieces. What do you notice about each piece? Measure and record the length of each piece.
3. Measure the heights of the children in your group with string. Hang the strings on the wall in order of height and label them.
4. Choose a measuring tool to measure the distance from your classroom to the office. Estimate the distance and write down your estimate. Measure and record.

# Exemplar 3

## A thematic approach to mathematics: The Olympic Games

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<b>Theme:</b>	The Olympic Games
<b>Class:</b>	Fifth or sixth
<b>Linkage:</b>	<i>Number, Shape and space, Measures: Length, Area, Weight, Time, Money, Data</i>
<b>Integration:</b>	English, SESE, Arts education, Physical education, SPHE
<b>Materials:</b>	worksheets, newspapers, encyclopaedias, string, ruler, compass, stop-watch, protractors, trundle-wheel, thermometer
<b>Implementation:</b>	whole class, group, paired or individual work on worksheets, collecting and compiling information, watching events on television at home

### The Olympic Games: Suggestions for developing a worksheet. *Measures: Time*

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1. What is the time difference between Sydney and Dublin?
2. The marathon starts at 9:00 a.m. in Sydney. The last competitor comes in at 12:30 p.m. What time did the race start and finish in Irish time?
3. What is the cost per minute of daytime and off-peak phone calls to Sydney? Find the total cost of five daytime phone calls and five off-peak calls lasting 20 minutes each.

### The Olympic Games: Suggestions for developing a worksheet. *Data*

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1. Examine a flight timetable to Sydney. How many flights leave Dublin before midday? How many flights arrive in Sydney before 18:00 hours Australian time?
2. Note and record the temperature in the stadium at 13:00 hours for seven days. Draw a graph to show the results. Record the temperature in Ireland at 13:00 hours for the same seven days. Compare and discuss the results.
3. Collect and record results, for example weights lifted, running speeds, heights and lengths of jumps.

Further worksheets on this theme can be devised for *Shape and space, Number, Measures: Weight, Length, Area, Money and Data*.

### Examples

1. Find the total weight of the first and last lift of six of the weight-lifters. What was the average weight of the competitors in the heavyweight event? (Calculators can be used when working with large numbers.) What is the heaviest weight you think you could lift? Try it out, but be careful!
2. What currency is used in Australia? What would it cost in Australian currency to fly from Dublin to Sydney?
3. The diameter of the discus pad is 2.5 metres. Find the radius. Find the circumference.
4. What is the best angle to throw a javelin at so that it travels the longest distance? Try throwing a bamboo pole and measure the distances thrown.
5. Collect the winning times of the qualifiers in each heat of the 100 metres sprint. Find the average time taken to run the course.

## Suggestions for linkage: The Olympic Games

### Measures

- **Length:** long jump, height of basketball players, high jump; length of running tracks, length and width of swimming lanes, journey from Ireland
- **Area:** area of playing courts
- **Weight:** weights of competitors and weights lifted, shot-put weights
- **Time:** time zones, times of events at the games and times here, time taken to travel to venue by plane, ship, Concorde, timetables for flights, events, 24-hour clock, temperatures, average speed
- **Money:** currencies, cost of phone calls, postage

### Shape and space

- angle of javelin, hammer, discus throws
- shape of running track
- draw arenas to scale
- use shapes to create flags and symbols

### Number

(all aspects of number can be used)

- winning times, winning margins
- numbers of participants from each country and in each event
- percentage of medals won by a country
- populations of countries
- fractions: design flags, examine existing flags

## Olympic Games

Linking areas of mathematics

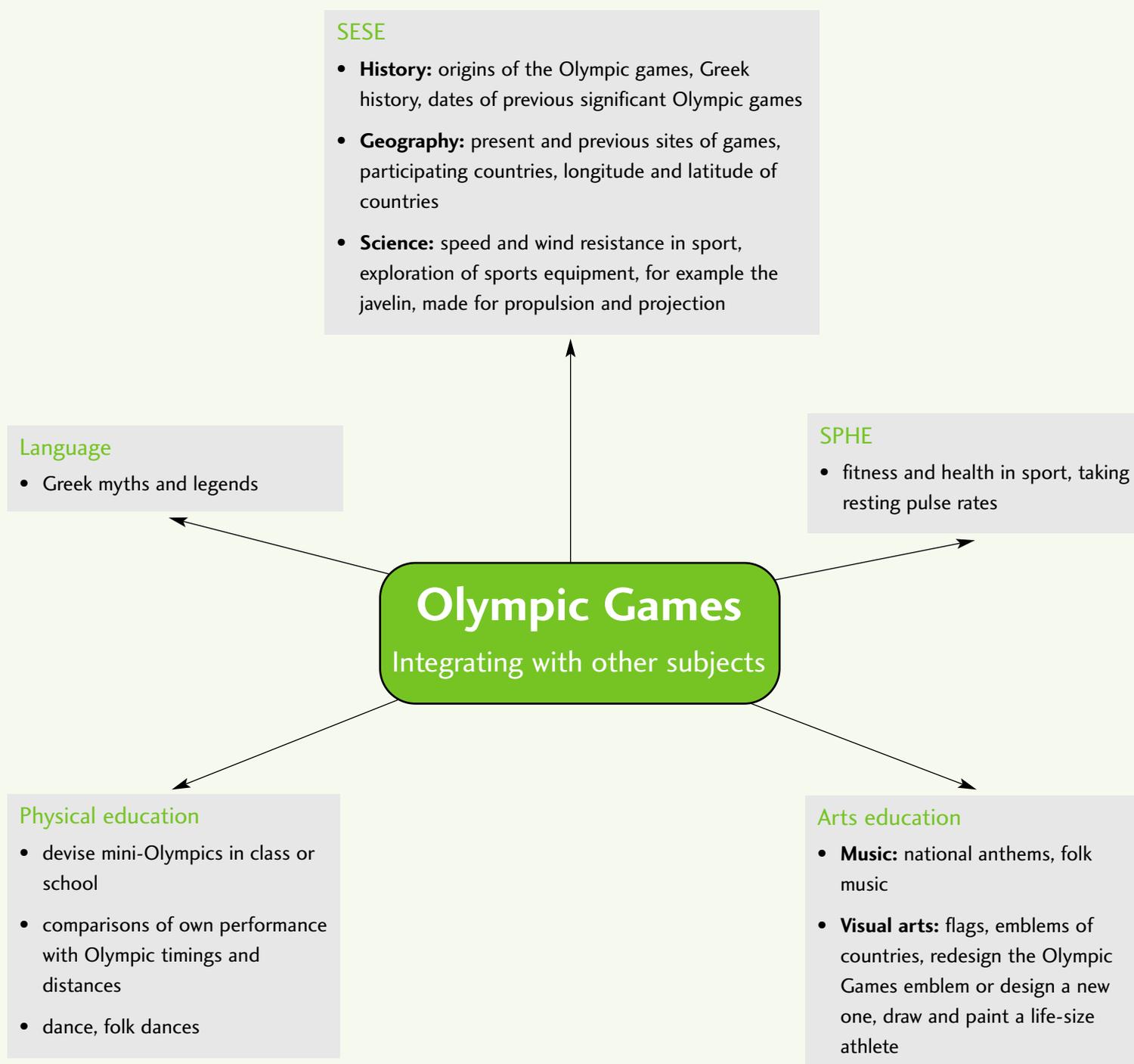
### Vocabulary

- prefixes: bi- (biped, bicycle), decimal, tri- (triathlon, triple jump, tripod), quad- (quadrennial, quadruped)
- heptathlon, decathlon, marathon
- Olympic, Olympia, Olympiad

### Data

- data collection and manipulation
- comparison of results with previous Olympic games
- averages
- bar charts, pie charts, graphs of results

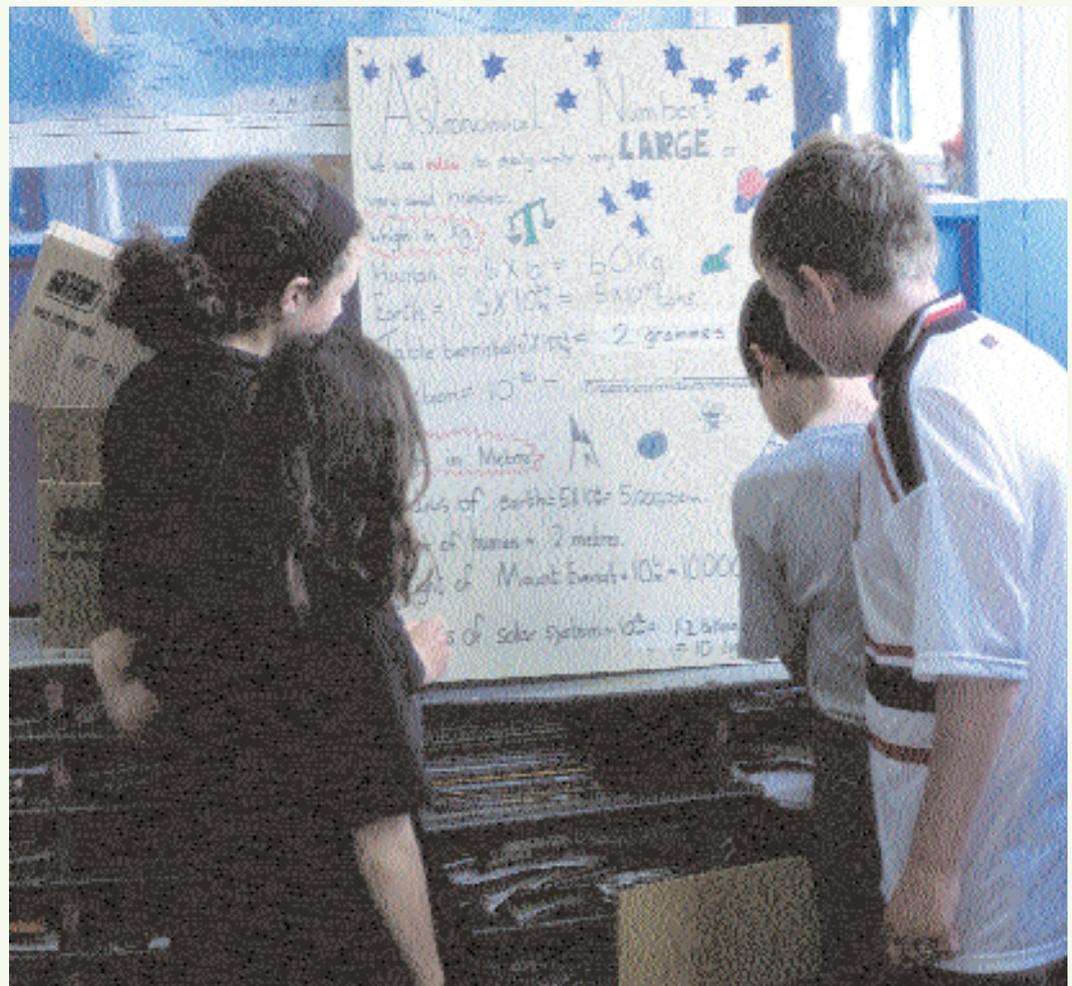
## Suggestions for integration: The Olympic Games



## Integration—suggestions for developing worksheets

1. Design a new logo and uniform for the Irish Olympic team.
2. Listen to the national anthems of various countries and compare the rhythms.
3. Time yourself running 100 metres. Compare your time with the winning time in the Olympic 100 metres.
4. Choose and learn a folk dance of a participating country.
5. Name the cities in which the 1988, 1992 and 1996 Olympics were held. Record the longitude and latitude of those cities.
6. What are the component events in the heptathlon and the decathlon?
7. Research the history of the ancient Olympic games. Find out about Greek myths and legends. What was life like for a young boy or girl in ancient Greece? Research and write about a hero or heroine of the modern Olympics.

This approach can be adapted to other sporting events or can be used with themes other than sport.



Children are fascinated by large numbers.

# An example of cross-strand planning for first class

	Month 1	Month 2	Month 3	Month 4
Week 1	<p><b>Strand: Measures</b> revision of measuring activities</p> <p><b>Strand: Number</b> oral and practical revision 0–20</p>	<p><b>Strand: Shape and space</b> oral and practical revision</p> <p><b>Strand: Shape and space</b> Strand unit: Spatial awareness</p>	<p><b>Strand: Number</b> Strand unit: Place value</p> <ul style="list-style-type: none"> <li>explore, identify and record place value 0–20</li> <li>add numbers without renaming 0–20</li> </ul> <p><b>Strand: Measures</b> Strand unit: Time oral and practical revision</p>	<p><b>Strand: Number</b> Strand unit: Place value reinforcement 0–20</p> <p>Strand unit: Addition</p> <ul style="list-style-type: none"> <li>number sentences, stories, problems 0–12</li> <li>without and with renaming 0–20</li> <li>mental strategies for addition 0–12</li> </ul> <p><b>Strand: Measures</b> Strand unit: Money oral and practical revision</p>
Week 2	<p><b>Strand: Number</b> Strand unit: Counting and numeration</p> <ul style="list-style-type: none"> <li>count objects in a set 0–30</li> <li>read, write and order numerals 0–30</li> </ul>	<p><b>Strand: Number</b> Strand unit: Place value</p> <ul style="list-style-type: none"> <li>explore, identify and record place value 0–16</li> </ul> <p><b>Strand: Shape and space</b> Strand unit: Spatial awareness</p>	<p><b>Strand: Number</b> Strand unit: Place value</p> <ul style="list-style-type: none"> <li>order sets of objects by number</li> <li>use the language of ordinal number</li> </ul> <p>Strand unit: Addition</p> <ul style="list-style-type: none"> <li>explore, develop and apply commutative, associative and zero properties</li> </ul>	<p><b>Strand: Number</b> Strand unit: Place value</p> <ul style="list-style-type: none"> <li>explore, identify and record 0–25</li> </ul> <p>Strand unit: Addition</p> <ul style="list-style-type: none"> <li>add numbers 0–25</li> <li>develop mental strategies for addition facts 0–14</li> </ul> <p><b>Strand: Measures</b> Strand unit: Money</p> <ul style="list-style-type: none"> <li>coins to 20p</li> </ul>
Week 3	<p><b>Strand: Number</b> Strand unit: Counting and numeration</p> <ul style="list-style-type: none"> <li>reinforce above work</li> <li>estimate number of objects in a set 0–12</li> </ul> <p>Strand unit: Place value</p> <ul style="list-style-type: none"> <li>explore, identify and record place value 0–13</li> </ul>	<p><b>Strand: Number</b> Strand unit: Place value</p> <ul style="list-style-type: none"> <li>reinforce above work 0–16</li> </ul> <p>Strand unit: Addition</p> <ul style="list-style-type: none"> <li>combine and partition sets 0–12</li> </ul>	<p><b>Strand: Number</b> Strand unit: Place value</p> <ul style="list-style-type: none"> <li>reinforce above work 0–20</li> </ul> <p>Strand unit: Addition</p> <ul style="list-style-type: none"> <li>add numbers without and with renaming 0–20</li> <li>develop mental strategies for addition facts 0–12</li> </ul>	<p><b>Strands: Number/Algebra/Shape and space/Measures</b> oral and practical revision</p>
Week 4	<p><b>Strand: Measures</b> Strand unit: Length oral and practical revision</p>	<p><b>Strand: Measures</b> Strand unit: Length</p> <ul style="list-style-type: none"> <li>estimate, compare, measure and record length using non-standard units</li> </ul>	<p><b>Strand: Algebra</b> oral and practical revision</p> <p>Strand unit: Exploring and using patterns</p> <ul style="list-style-type: none"> <li>odd and even 0–12</li> </ul>	

# Using technology

## Calculators can

- help in the development of an understanding of the four rules of arithmetic and their interrelationships
- help with problem-solving by focusing on higher-level skills
- give the child confidence to try more difficult mathematical tasks by removing computational barriers
- allow the child to explore the number system and to discover facts and relationships
- be used to create patterns, predict and check results and explore aspects of number structure

## Calculators

Calculators can help in the development of problem-solving skills by allowing the child to focus on the structure of a problem and possible methods of solution. The skills of estimation and trial-and-error methods of problem-solving can be developed. This allows children to look at problems from the real world, where the numbers involved might otherwise be too unwieldy for them to handle.

The child's attitude towards mathematics can also benefit from the use of the calculator by taking some of the drudgery out of complex calculations. Children can then experience success in mathematics, which is an important factor in maintaining interest and enthusiasm for the subject. By making tasks more manageable, children may gain the confidence to try new tasks and persevere with problems that cannot be solved quickly. Exploration and investigation can be encouraged and discussion promoted. Calculators can make problem-solving more accessible to low-achieving children who might otherwise never experience correct problem solution because of the frequency of computational errors.

The calculator can play a significant role in the mathematics programme from fourth to sixth class. However, children will need to know and use number facts to check the reasonableness of answers obtained with the calculator. They will also need to have a good understanding of number in order to make judgements about when it is appropriate to estimate, to calculate mentally or to make a quick pencil-and-paper calculation. The ability to decide when it is necessary to use a calculator is equally important.

It is recognised that some parents and teachers will be unfamiliar with the ways in which calculators can be used to develop mathematical skills and understanding. Parents will need to be reassured that their children will continue to master number facts and processes.

## Choosing calculators for the classroom

Many children will have experimented with calculators at home, and many of them will own one. However, if they are to be used in the classroom, it is advisable that the children all have access to the same type of calculator.

- keys on classroom calculators should be of a reasonable size and have a positive click action to avoid keying in incorrect numbers

Calculators are tools to use when appropriate. Children must be able to ask themselves:

Can I do it in my head?

Is it easier to do on paper?

How reasonable is the answer?

Do I really need to use the calculator?

- the display should have at least eight digits and be large enough for two or three children to see
- a constant memory function is useful when building up number patterns.

Some calculators operate by using *arithmetic logic*. These allow operations to be performed in the sequence in which they are keyed in: for example  $3 + 4 \times 2$  will give 14 as a result.

Calculators that use *algebraic logic* wait until the whole equation is keyed in before commencing the operation, hence  $3 + 4 \times 2$  will give 11 as a result, as it will do the multiplication operation first. Algebraic logic uses priorities in sequences of operation which we call BOMDAS (brackets, of, multiplication, division, addition and subtraction).

Interesting experiments can be undertaken by comparing the results of both types of calculator.

Using calculators in the classroom

- children should be taught how to handle and take care of calculators
- they should be given adequate time to experiment with them
- they will need to understand the priority of operations (BOMDAS) to develop efficient and accurate use of the calculator

- it is very important that children are taught to analyse the problem in order to assess which operation is required. They should also be comfortable with estimation so that they will realise when a calculation may be incorrect.

## Information and communication technology

Like the calculator, the computer is a tool that can be used by children and teachers but is not a substitute for good teaching.

*Drill and practice* programs can provide an attractive alternative to pen-and-paper tasks and is particularly useful with younger or less-able children who may need stimulation or extra practice on a particular topic. There are many useful programs available that can be used with infants to develop pattern recognition, shape matching and early number work but such use of the computer is short-lived and must not be over-used when a child has achieved mastery.

*Adventure programs* often require the child to solve specific mathematical problems in order to progress through a game, and they also give experience in the area of general problem-solving. Many of these programs allow a choice of level so they can be used with different classes.

### Some uses of ICT in mathematics

- drill and practice
- adventure programs
- databases
- spreadsheets
- LOGO
- using the internet to access materials and information

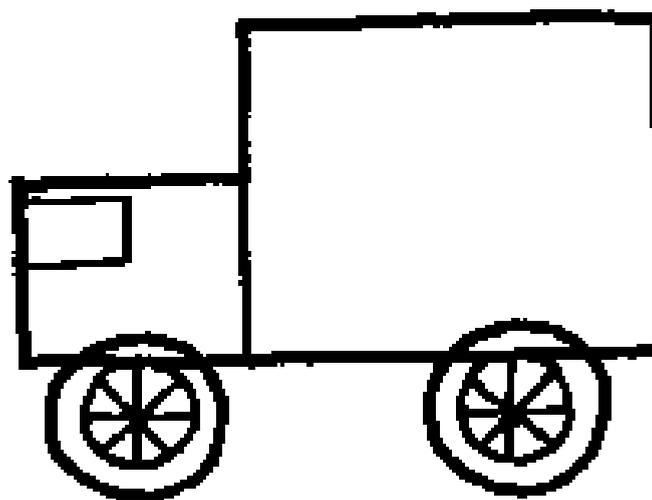
*Databases* are perhaps one of the most effective areas of ICT use in schools. We are constantly being presented with results of surveys, charts and forecasts in the media. Simple databases give children the opportunity to manipulate smaller amounts of data relevant to their own interests, for example height, weight or colour of eyes. The use of more sophisticated data handling programs will enable children to develop



this concept in handling larger amounts of information, perhaps on a school level; for example, have all tall children blue eyes? how many children have a cat for a pet? how many boys have cats? Data handling is also important in other areas of the curriculum and can be used to investigate population trends or rainfall amounts in geography, or to collate information in a local history project.

*Spreadsheets* are becoming increasingly popular for exploration of number patterns and for tabulating numerical data. They are also useful for teachers in keeping numerical records that need to be updated on a regular basis, for example reading ages.

LOGO is a *computer language* that has been used successfully with children for many years. It is extremely valuable in work on direction, line, angles and shape. It also encourages the re-working of a problem and the concept of multiple approaches to solving a problem. LOGO requires logical thought, and this can be helpful to a child who finds it difficult to deal with large, complex word problems.



The child can plan his/her work on squared paper, discuss possible approaches and then refine them. When he/she is satisfied with the 'plan' it can be entered into the computer. It can then be tried out and further refined. The off-computer planning is an extremely important part of the process. This approach works best when children work in pairs and are encouraged to plan, discuss, record and test their plan. The emphasis will always be on the process rather than the product. When children work together like this it encourages collaboration and co-operative thinking.

*Internet access.* The internet can be used to access information and data that can be used by the children in problem-solving or data handling exercises. It can also be used by teachers to download materials and information relevant to their work.

As with calculator use, children must become more discerning about when it is appropriate to use ICTs to solve a problem and which element of that technology will be of most use to them.

# Looking at children's work

## Children and assessment

Children should be comfortable with different assessment methods. They need to understand the need to have the correct equipment, the importance of silence and be taught to develop test-taking skills. These skills include listening, analysis of a problem so they can define what it is they are being asked to do, form-filling techniques and presentation skills. It is important that children view assessment as a positive experience which can help them in future work. They can be encouraged to take an active part in recording their own successes in a personal notebook and to practise self-assessment by discussing their achievements or problems with the teacher.

### *Teacher observation*

The teacher observes that one child may prefer to work quietly in a corner while another may excel in group situations. He/she observes the child who still needs to use fingers for counting, while another may prefer to use a number line. Some children may be poor readers but can perform mental calculations quickly or suggest creative solutions to problems. Such incidental observations give the teacher an overview of the individual's mathematical development and have an important contribution to make to the overall pupil profile. They also provide the teacher with information on the child's attitudes to the subject.

It is not possible to document every detail of the child's behaviour or responses, but short comments that are written into a notebook can assist the teacher in planning the next stage of the work. Such information can also be conveyed to parents so that they can help the child with homework.

### *Teacher-designed tasks and tests*

Teachers set tasks for children in every mathematics lesson. They provide workcards and worksheets and set problems for the children to solve. Many of these tasks are in themselves a form of assessment. When a child is asked to measure a table or the cover of a book the response indicates to the teacher whether that child has acquired the concept of length. Tests that focus on a unit of work just completed help the teacher to evaluate what should be done next.

### *Work samples, portfolios and projects*

It is very useful to have a folder in which samples of work are kept. The child can play a part in the selection process by choosing a piece of work of which he/she is proud. Comments can be written on such sheets by the teacher to add to the overview of the child's mathematical development. Completed worksheets and copies can also be added, as they provide a concrete example of the child's work. The manageability of such a folder must be considered, as storage could be a problem. Only some samples of the

child's work need be included and results of teacher-made tests, mastery records and curriculum profiles can also be included.

#### *Curriculum profiles and mastery records*

Curriculum profiles itemise the development of skills in the child: for example, does he/she choose the correct measuring instrument for a task or can he/she solve two-step problems? Mastery records examine more specific areas of mathematics, for example the addition of two-digit to two-digit numbers.

#### *Diagnostic testing*

Diagnostic testing is often done by a remedial teacher. The analysis of work done by the child also has a diagnostic role. If the child is particularly good at problem-solving it could indicate that he/she needs more challenging work. Analysis of the errors a child makes can assist the teacher in planning the next unit of work for that child.

#### *Standardised testing*

Norm-referenced and criterion-referenced tests are useful in obtaining an overview of the child's mathematical development. They can be given to a whole class and provide accurate information that can be added to the pupil profile. More detailed information on these forms of testing is given on p. 119 of the curriculum.

#### *Pupil profiles*

These profiles are a compilation of the results gathered from the administration of different assessment tools. They provide useful information for reporting to parents or other teachers.



Section 6

# Appendix

# Overview of skills development

	Applying and problem-solving	Communicating and expressing	Integrating and connecting	Reasoning	Implementing	Understanding and recalling
<b>Infant classes</b>	<ul style="list-style-type: none"> <li>select appropriate materials and processes for mathematical tasks</li> <li>select and apply appropriate strategies to complete tasks or solve problems</li> <li>recognise solutions to problems</li> </ul>	<ul style="list-style-type: none"> <li>discuss and explain mathematical activities</li> <li>record results using diagrams, pictures and numbers</li> <li>discuss problems presented pictorially or orally</li> </ul>	<ul style="list-style-type: none"> <li>connect informally acquired mathematical ideas with formal mathematical ideas</li> <li>recognise mathematics in the environment</li> <li>recognise the relationship between concrete, verbal, pictorial and symbolic modes of representing numbers</li> <li>carry out mathematical activities which involve other areas of the curriculum</li> </ul>	<ul style="list-style-type: none"> <li>classify objects into logical categories</li> <li>recognise and create sensory patterns</li> <li>justify the processes/results of activities</li> </ul>	<ul style="list-style-type: none"> <li>devise and use mental strategies/ procedures for carrying out mathematical tasks</li> <li>use appropriate manipulatives to carry out tasks and procedures</li> </ul>	<ul style="list-style-type: none"> <li>understand and recall terminology</li> </ul>
<b>First and second classes</b>	<ul style="list-style-type: none"> <li>select appropriate materials and processes for mathematical tasks/applications</li> <li>apply concepts and processes in a variety of contexts</li> </ul>	<ul style="list-style-type: none"> <li>listen to and discuss other children's descriptions/ explanations</li> <li>discuss and record using diagrams, pictures and symbols</li> </ul>	<ul style="list-style-type: none"> <li>understand the mathematical ideas behind the procedures he/she uses</li> </ul>	<ul style="list-style-type: none"> <li>make guesses and carry out experiments to test them</li> <li>recognise and create mathematical patterns and relationships</li> </ul>	<ul style="list-style-type: none"> <li>execute procedures efficiently</li> </ul>	<ul style="list-style-type: none"> <li>understand and recall terminology and facts</li> </ul>

	Applying and problem-solving	Communicating and expressing	Integrating and connecting	Reasoning	Implementing	Understanding and recalling
<b>Third and fourth classes</b>	<ul style="list-style-type: none"> <li>select appropriate materials and processes for mathematical tasks and applications</li> <li>analyse problems and plan an approach to solving them</li> <li>select and apply a variety of strategies to complete tasks/projects or solve problems</li> <li>evaluate solutions to problems</li> </ul>	<ul style="list-style-type: none"> <li>discuss and explain the processes used or results of mathematical activities/projects/problems</li> <li>discuss and record processes and results using a variety of methods</li> <li>discuss problems presented orally, pictorially or diagrammatically; carry out analyses</li> </ul>	<ul style="list-style-type: none"> <li>connect informally acquired mathematical ideas and processes with formal mathematical ideas and processes</li> <li>understand the connections between mathematical procedures and concepts</li> <li>represent mathematical ideas and processes in different modes: verbal, pictorial, diagrammatic, symbolic</li> <li>recognise and apply mathematical ideas and processes in other areas of the curriculum</li> </ul>	<ul style="list-style-type: none"> <li>make hypotheses and carry out experiments to test them</li> <li>make informal deductions involving a small number of steps</li> <li>explore and investigate mathematical patterns and relationships</li> <li>reason systematically in a mathematical context</li> </ul>	<ul style="list-style-type: none"> <li>execute standard procedures efficiently with a variety of tools</li> </ul>	<ul style="list-style-type: none"> <li>understand and recall terminology, facts and definitions</li> </ul>
<b>Fifth and sixth classes</b>	<ul style="list-style-type: none"> <li>reflect upon and evaluate solutions to problems</li> </ul>	<ul style="list-style-type: none"> <li>discuss and explain processes and results in an organised way</li> <li>discuss problems and carry out analyses</li> </ul>		<ul style="list-style-type: none"> <li>search for and investigate mathematical patterns and relationships</li> </ul>		<ul style="list-style-type: none"> <li>understand and recall facts, definitions and formulae</li> </ul>

# Symbols, numerals, fractions and terminology

	Junior infants	Senior infants	First class	Second class	Third class	Fourth class	Fifth class	Sixth class
<b>Symbols</b>		+ =	- frame □ p cent	< > £ €	× ÷ decimal point		% degree°	positive and negative notation +2, -5 exponent e.g. 4 <sup>2</sup>
<b>Numerals</b>	0–5	6–10	to 99	to 199	to 999	to 9999		
<b>Decimals</b>					one place of decimals 0.1 $\frac{1}{10}$	two places of decimals 0.01 $\frac{1}{100}$	three places of decimals 0.001 $\frac{1}{1000}$	
<b>Fractions</b>			$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$ $\frac{1}{10}$	$\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{9}$ $\frac{1}{12}$		
<b>Measures</b>			metre litre kilogram	centimetre	gram millilitre	kilometre m <sup>2</sup> , cm <sup>2</sup>	millimetre	ares hectares
<b>2-D shapes</b>	square, circle, rectangle, triangle		semi-circle	oval	hexagon	parallelogram rhombus pentagon octagon triangles: <i>equilateral</i> <i>isosceles</i> <i>scalene</i>	quadrilateral trapezium	
<b>3-D shapes</b>		cube, cuboid sphere, cylinder		cone	triangular prisms pyramid		tetrahedron	octahedron
<b>Time</b>	vocabulary of time	read time in one-hour intervals	read time in half-hour intervals	read time in quarter-hour intervals	read time in five-minute intervals		24-hour clock	



# Suggested list of mathematical equipment

## Number

- number lines, strips, abacus and rubber stamp abacus
- magnetic board strips
- counters, beads, string, buttons, Unifix cubes, spools and sorting trays
- Dienes blocks, Cuisenaire rods
- pegboards and pegs
- number ladder
- story of 10 boards
- hundred square (with and without numbers)
- fraction, decimal, percentage walls
- number slabs
- number balance
- playing-cards and dominoes
- notation boards

## Shape and space

- 2-D and 3-D shapes, geo-boards, tangrams, geo-strips
- direction compass
- set-squares, clinometer
- blackboard compass, set-squares and protractor
- 360° and 180° protractors
- gummed paper, paper shapes
- construction straws
- construction kits

## Measures

### (standard and non-standard)

#### Length

- unmarked sticks, metre stick, half and quarter-metre sticks, trundle-wheel, height chart, tape measures, rulers, ribbon or string
- bamboo poles

#### Weight

- balance, kitchen scales and bathroom scales, weights, spring balance

#### Capacity

- litre, half and quarter-litre containers, varied collection of containers for comparison

#### Time

- clock faces and rubber stamps, clock (analogue and digital)
- calendar and date stamps
- sequencing pictures

#### Money

- facsimile money, money stamps

## General mathematical equipment

- Lego, books and games
- water or sand tray
- scissors (left and right-handed)
- magnifying glass, magnets, microscope
- rain gauge, barometer and thermometer
- overhead projector
- television and video programmes
- computer programs
- calculators
- selection of dice

# Source references for the curriculum and guidelines

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# Glossary

This limited glossary contains commonly understood working definitions of some mathematical terms.

algorithm	a logical, arithmetical or computational procedure that if correctly applied ensures the solution of a problem
analogue clock	a clock on which hours, minutes and sometimes seconds are indicated by hands on a dial
are	a unit of area equal to 100 square metres
associative	an operation such as multiplication or addition is associative if the same answer is produced regardless of the order in which the elements are grouped, e.g. $(2 + 3) + 5 = 10$ , $2 + (3 + 5) = 10$
cardinal number	a number denoting quantity but not order in a set
commutative	giving the same result irrespective of the order of the elements in an operation, e.g. for addition and multiplication $6 + 2 = 8$ , $2 + 6 = 8$ , $5 \times 7 = 35$ , $7 \times 5 = 35$
composite number	a number with more than two factors that is not a prime number, e.g. 6, 10
denominator	the divisor in a fraction
diameter	a straight line connecting the centre of a circle with two points on the perimeter
distributive	the same result is produced when multiplication is performed on a set of numbers as when performed on the members of the set individually, e.g. $5 \times 4 = (3 + 2) \times 4 = (3 \times 4) + (2 \times 4)$
dividend	a number or quantity to be divided <i>by</i> another number or quantity
divisor	a number or quantity to be divided <i>into</i> another number or quantity
equation	a mathematical sentence with an equals sign
hectare	a unit of area equal to 100 ares

line symmetry	a shape has line symmetry if one half of the shape can be folded exactly onto the other half
number sentence	an equation or statement of inequality, e.g. $4 + x = 11$ , $4 \times 2 < 12$ or $2 + 5 = 7$
numerator	the number above the line in a fraction
ordinal number	a number denoting relative position in a sequence, e.g. first, second, third
perimeter	the sum of the length of the sides of a figure or shape
prime factor	a factor that is a prime number
prime number	a whole number that has only two factors, itself and 1, e.g. 2, 3, 7
product	the result of multiplying two numbers
quotient	the result of dividing one number by another number
radius	a line from the centre of a circle to a point on the circumference; a radius is half of the diameter
ratio	the relationship between two numbers of the same kind; e.g. the ratio of 2 kg to 6 kg is 2 : 6
subitise	tell at a glance, without counting, the number of items in a set
subtrahend	the number to be subtracted from another number, e.g. $10 - 4$ (4 is the subtrahend)
tessellation	shapes tessellate if they fit together exactly, form a repeating pattern and make an angle of $360^\circ$ at the points of contact
variable	a letter or symbol that stands for a number, for example $y + 7 = 12$

# Membership of the Curriculum Committee for Mathematics

These guidelines have been prepared under the direction of the Curriculum Committee for Mathematics established by the National Council for Curriculum and Assessment.

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