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Department of Education

Primary Mathematics Curriculum

For Primary and Special Schools

Prepared by the National Council for Curriculum and Assessment (NCCA)

Contents

1. Introduction

The primary curriculum supports high-quality learning, teaching and assessment for all children attending primary and special schools. The curriculum recognises primary education as a time of 'being' and 'becoming' – highlighting the importance of interesting, relevant and appropriately challenging experiences. It is important that children enjoy and benefit from these experiences in the present, whilst simultaneously equipping them for learning in the years ahead. The primary curriculum is premised on a vision of children as unique, competent and caring individuals. It aims to provide a strong foundation for every child to thrive and flourish. It supports children in realising their full potential as individuals and as members of communities and society during childhood and into the future. This takes place through high-quality learning, teaching and assessment that is inclusive and responsive.

The primary curriculum acknowledges that from birth, children begin their educational journey through interactions and experiences with the world around them. In primary and special schools, children engage in playful and engaging learning experiences that build upon the knowledge, skills and dispositions they have acquired at home and in preschool settings through *Aistear: the Early Childhood Curriculum Framework*. As children progress through primary and special school their learning connects with, and is further progressed through, the learning experiences provided in Junior Cycle. Each child's learning journey is different, and so the curriculum provides flexibility and choice to teachers and school leaders as they support children in their holistic development.

Principles of learning, teaching and assessment

The following principles, as outlined in the *Primary Curriculum Framework*, convey what lies at the heart of primary education, including children's learning of Mathematics.

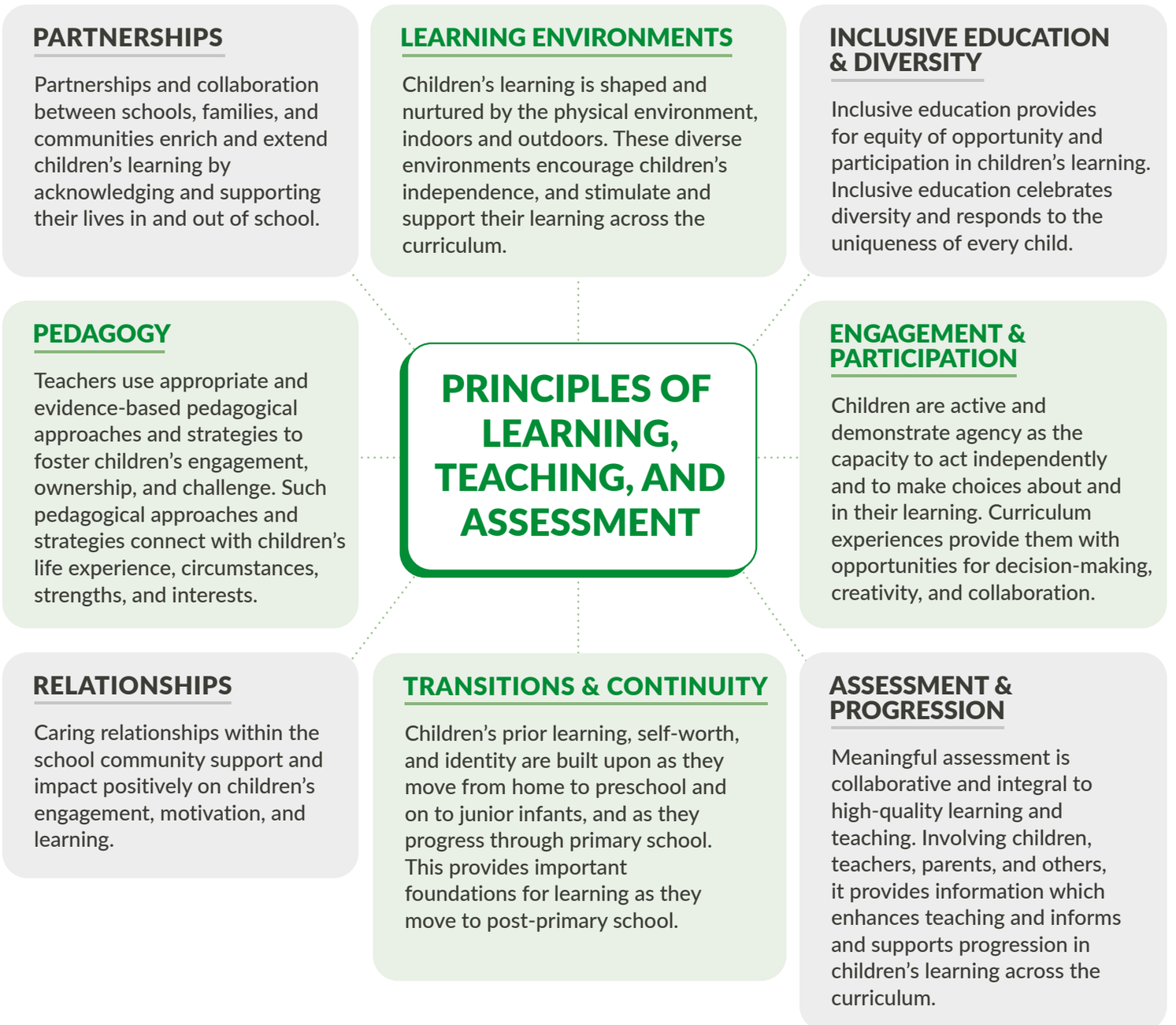

Figure 1: Principles of learning, teaching, and assessment

Children's mathematical learning experiences

In the primary curriculum, Mathematics is situated within the Science, Technology, Engineering and Mathematics (STEM) Education curriculum area. Mathematics provides an important foundation upon which to develop and refine children's learning in STEM Education. Rich learning experiences in STEM Education help children to understand relationships, connections and patterns, and to engage fully with the world around them.

The primary curriculum has seven key competencies which overlap and combine to support the curriculum's vision. As outlined in the *Primary Curriculum Framework*, the competencies build on the capabilities children acquire through their early childhood education experiences with *Aistear: the Early Childhood Curriculum Framework*; and are further strengthened in post-primary school in Junior Cycle. As children work towards the Learning Outcomes in the Mathematics curriculum and engage in rich mathematical learning experiences, they simultaneously build and develop the key competencies.

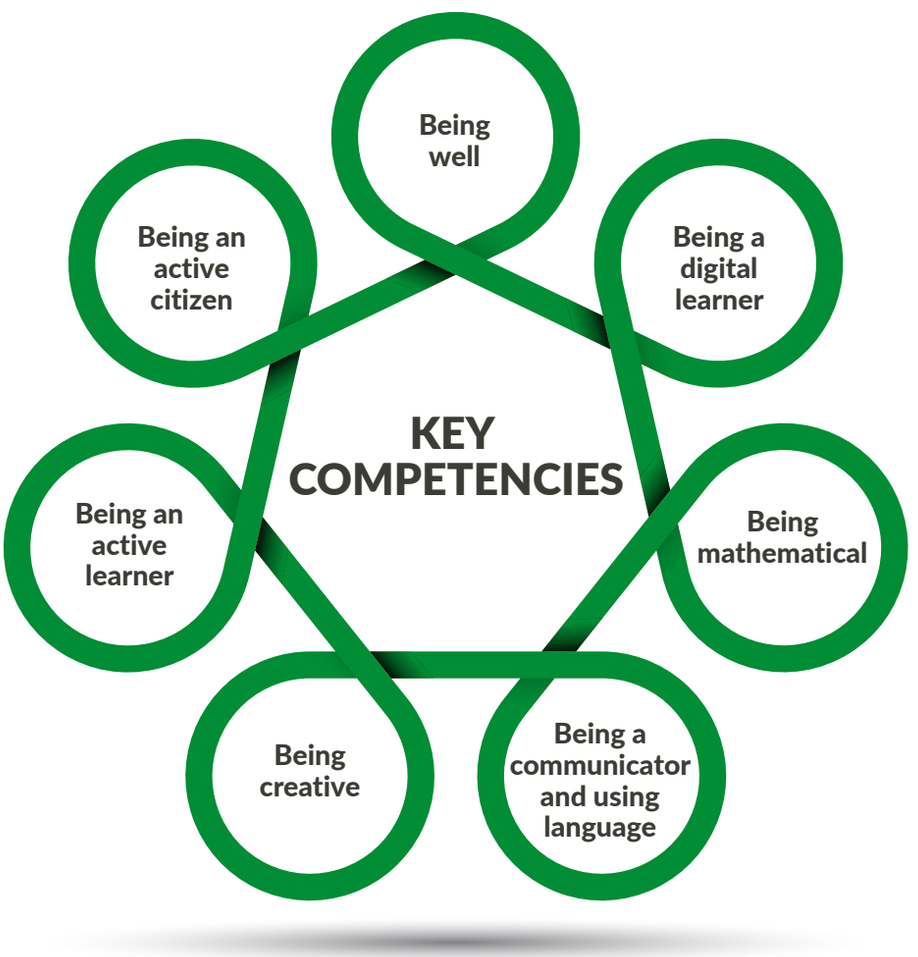

Figure 2: Key competencies

Table 1: Examples of attributes of each key competency developed through learning in Mathematics

Key Competency	Examples of attributes developed through learning in Mathematics
Being an active citizen	• Applying mathematical knowledge and skills to solve real world problems • Using Mathematics to generate and manage information that informs and promotes a critical understanding of the world and society
Being creative	• Exploring Mathematics with curiosity, open-mindedness and imagination • Investigating, using and sharing diverse mathematical ideas and solution paths
Being a digital learner	• Using a range of digital technologies to expand how to engage with, express and represent complex mathematical ideas • Reducing complexity and allowing for the development of higher-order thinking
Being mathematical	• Framing real world information and situations in mathematical terms • Engaging in mathematical activities that involve enjoyment, effort, risk-taking, critical thinking and reflection
Being a communicator and using language	• Expressing thinking using mathematical language, signs and symbols • Sharing and comparing ways of representing mathematical thinking and ideas
Being well	• Developing and contributing unique perspectives and ideas about mathematical situations • Applying Mathematics in meaningful contexts and experiencing learning success
Being an active learner	• Persevering with complex mathematical problems and tasks • Reflecting on and evaluating approaches and solutions to mathematical tasks

Overview of the Mathematics curriculum

The opening sections of the *Primary Mathematics Curriculum* present the Rationale, Aims, Strands and Elements, and Learning Outcomes. Chapter 6 provides guidance on the curriculum in practice, while the final chapters provide an overview of the Primary Mathematics Toolkit and a glossary of terms.

The *Primary Mathematics Curriculum* is supported by the Primary Mathematics Toolkit. It contains a range of supports for enacting the curriculum, such as mathematical concepts, progression continua, support materials and examples of children's learning.

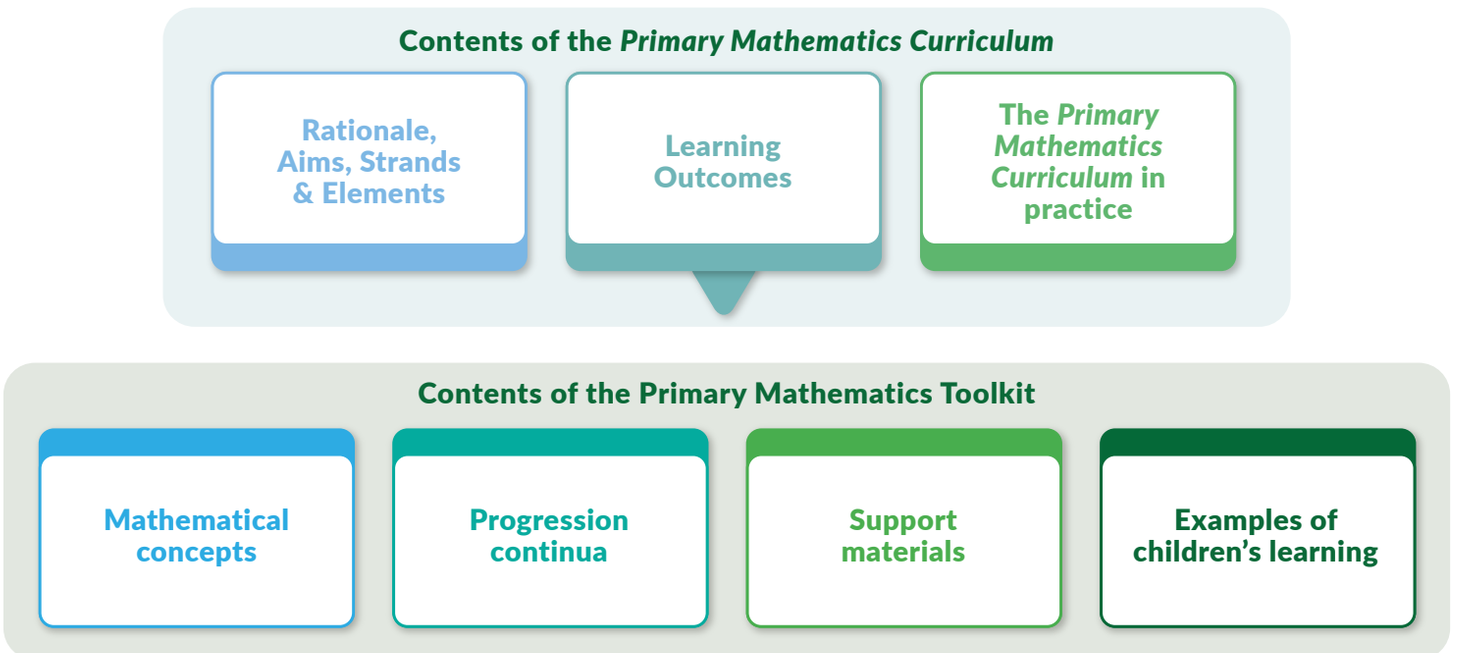

Figure 3: Overview of *Primary Mathematics Curriculum* and Toolkit


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+5=10
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2. Rationale

Learning Mathematics

Mathematics is the study of the relationships, connections and patterns that surround us, and is thus intrinsic to our concept of the world. Mathematics greatly enhances our capacity to understand and engage fully with the world around us. A child's mathematical learning journey begins from birth. Children initially learn Mathematics through their interactions and experiences in their home environment. They later build on this learning through early childhood, primary, special and post-primary education.

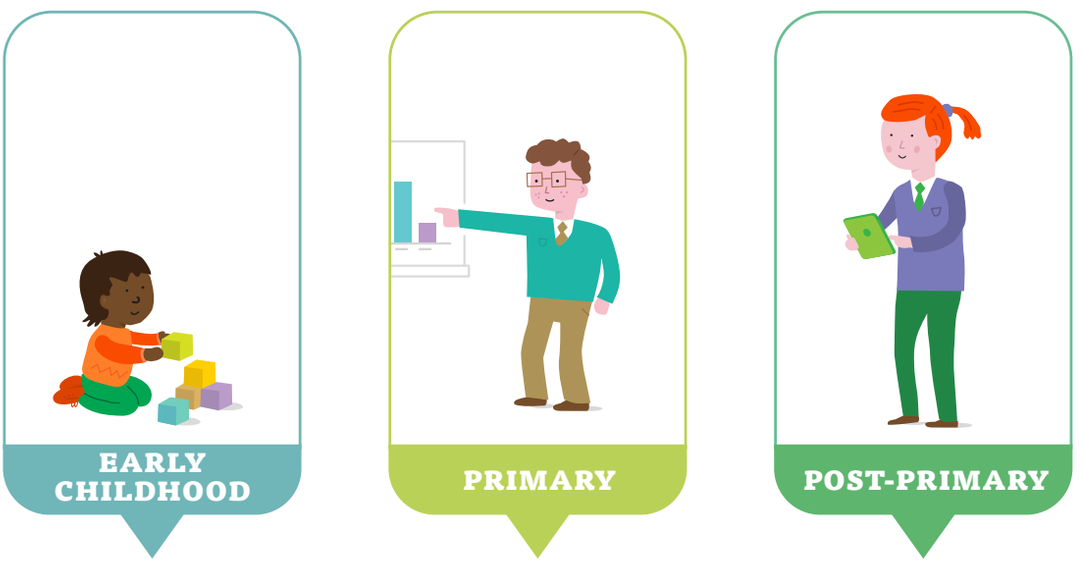


Figure 4: Building on children's learning in Mathematics

Every child is mathematical

Every child has an innate, intuitive and instinctive sense of Mathematics. Every child is capable of engaging with mathematical concepts and ideas from birth, and deepening and developing their learning over time.

Primary mathematics education evokes children's innate ability to think and communicate mathematically, to solve problems and to make sense of the world using Mathematics. Children are encouraged and supported to have a positive disposition to Mathematics and to develop their mathematical understanding, language, communication skills, perseverance and resilience, interactions and expressions.

Mathematics is both a human and social phenomenon

Mathematics learning is dependent on social and cultural experiences as well as on children's educational experiences in school.

Primary mathematics education provides children with playful and engaging learning experiences that promote mathematical thinking, such as modeling, thinking aloud and maths talk. It also provides opportunities for children to collaborate, communicate mathematical thinking and express their understanding in multiple ways and in various contexts.

Mathematics is a tool that helps us to make sense of our world

Mathematics is used to think about, see and organise our everyday lives and the world. Primary mathematics education equips children with mathematical, statistical and financial literacy skills and tools. It helps them to better function in, critically engage with and navigate the world around them. It also enables children to develop the language of Mathematics so that they can communicate and solve problems using Mathematics.

Mathematics is beautiful and worthy of pursuit in its own right

It is important that children have the opportunity to engage with Mathematics as a discipline in its own right and to explore its many intriguing aspects. Through playful, creative and engaging learning opportunities, children can experience the beauty and power of Mathematics. Primary mathematics education fosters a love of Mathematics. It provides children with the opportunity to explore, discover and refine their ideas. Children are supported to think critically and flexibly, and to be creative and innovative in their approach to learning Mathematics.

Mathematics is everywhere and for everyone

Mathematics is a human activity that develops in response to everyday problems and interactions. Primary mathematics education provides children with opportunities to engage with appropriately rich, meaningful and challenging Mathematics in educational settings, including social and familial settings. Such engagement results in children co-constructing knowledge and skills as they interact and collaborate to solve real and complex problems.

3. Aims

The over-arching aim of the *Primary Mathematics Curriculum* is the development of mathematical proficiency. Mathematical proficiency encompasses conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. Importantly, all five aspects are interwoven and interdependent.

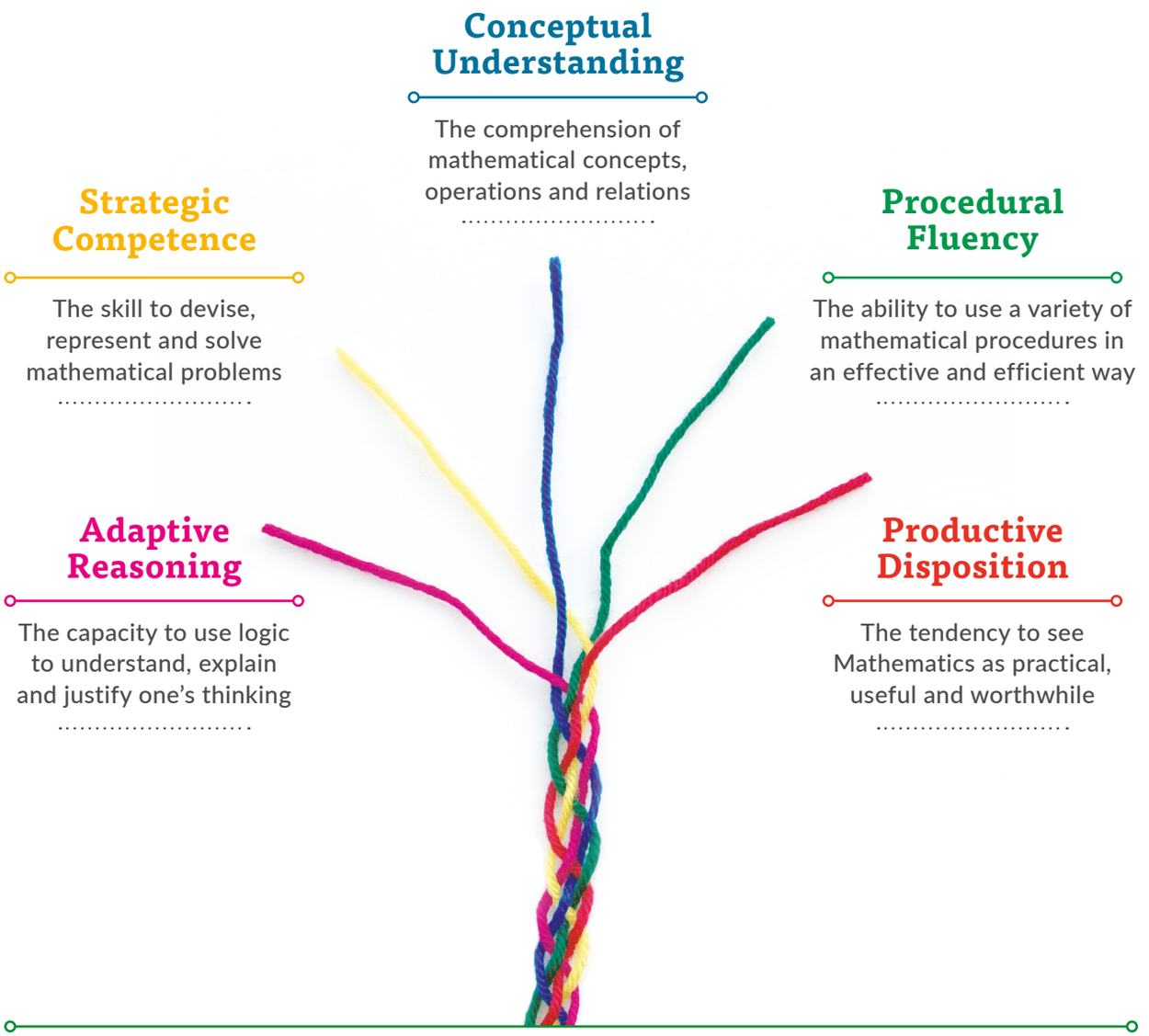


Figure 5: Five aspects of mathematical proficiency

4. Strands and Elements

Strands

Strands outline the main categories of mathematical learning (what children learn) across five domains or content areas of primary mathematics: algebra; data and chance; measures; number; and shape and space. Each strand has a set of strand units.

Table 2: Strands and strand units

Algebra	Data and chance	Measures	Number	Shape and space
Patterns, rules and relationships	Data	Measuring	Uses of number	Spatial awareness and location
Expressions and equations	Chance	Time	Numeration and counting	Shape
		Money	Place value and base ten	Transformation
			Sets and operations	
			Fractions	

Elements

Elements describe the main categories of processes (how children learn) that children engage in as they learn Mathematics. These processes include: connecting, communicating, reasoning, justifying, representing, problem-solving, generalising and argumentation, and are categorised into four elements: understanding and connecting; communicating; reasoning and applying; and problem-solving. These are central to the development of children's mathematical proficiency.

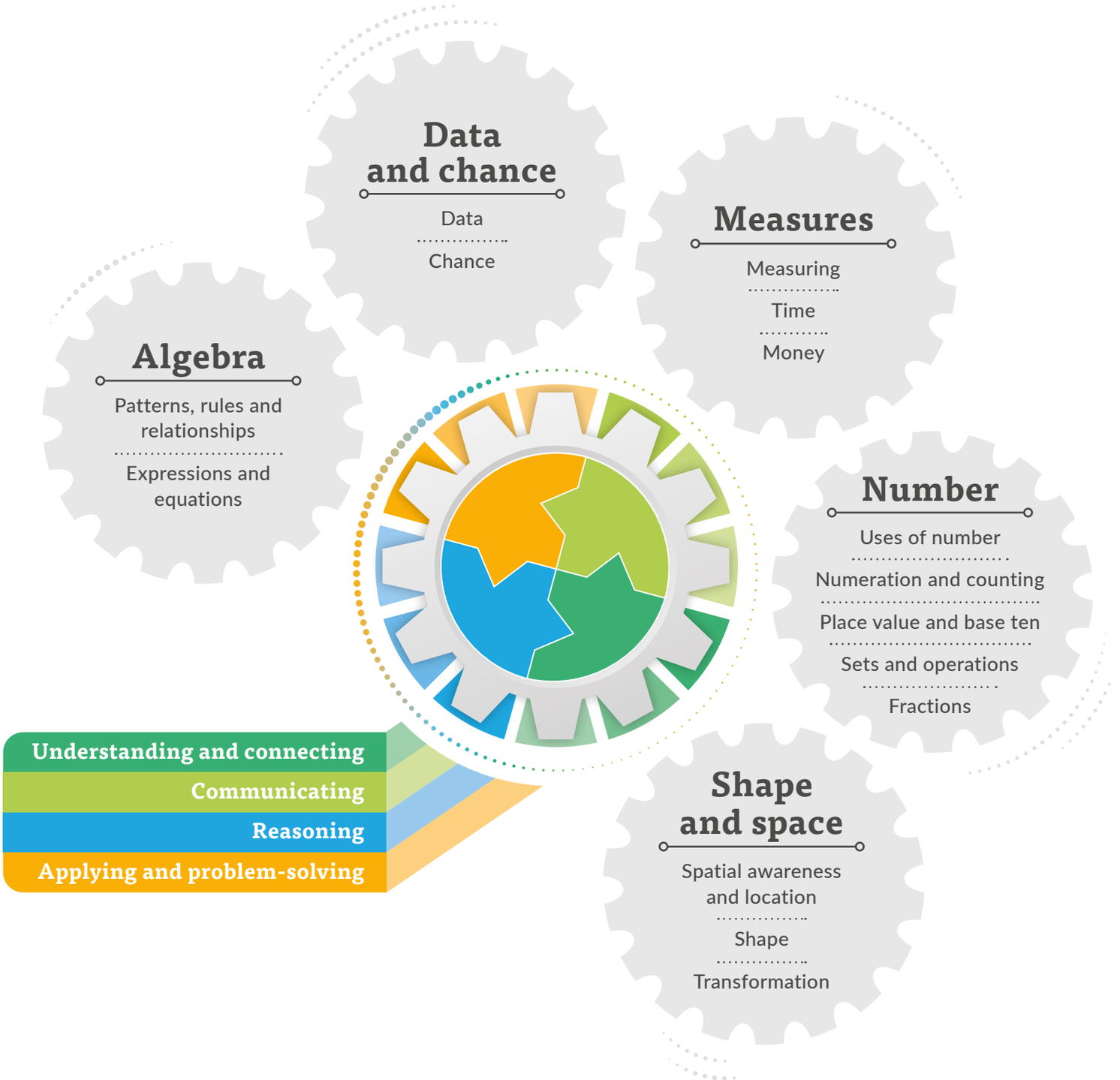


Figure 6: The strands and elements of the *Primary Mathematics Curriculum*

Element 1: Understanding and connecting	Children make connections between related concepts and procedures—the 'why' and the 'how' of Mathematics—and between new and prior knowledge, in order to make sense of what they are learning. They apply and connect their understanding to contexts within Mathematics, with other areas of learning and with the real world.
Element 2: Communicating	Children use appropriate language and/or means of communication and a variety of representations and conventions to convey thinking, ideas, relationships and logical arguments. They improve and refine their thinking and communication through engaging in inquiry-based learning and social learning environments that promote discourse and groupwork.
Element 3: Reasoning	Children develop and apply reasoning to make, assess and justify ideas and conjectures. They engage in logical thought and actions such as analysing, proving, inferring and generalising. They plan and construct solid arguments to justify their explanations, proofs and decision making.
Element 4: Applying and problem-solving	Children investigate, develop, select, apply, interpret, model and compare a variety of problem-solving situations and strategies as they explore Mathematics and deepen their mathematical understanding. They apply their mathematical knowledge and skills in flexible, efficient and creative ways to solve problems; conduct investigations; and develop and share their computational thinking.

Connections across strands

The strand structure of the curriculum should not be taken to imply that topics are to be explored in isolation. Where appropriate, connections should be made between and across the strands, and with other areas of learning, to enhance their interrelatedness and relevance for learning. While there are numerous connections that can be made between strands, the following graphic (Figure 7) highlights a number of these connections.

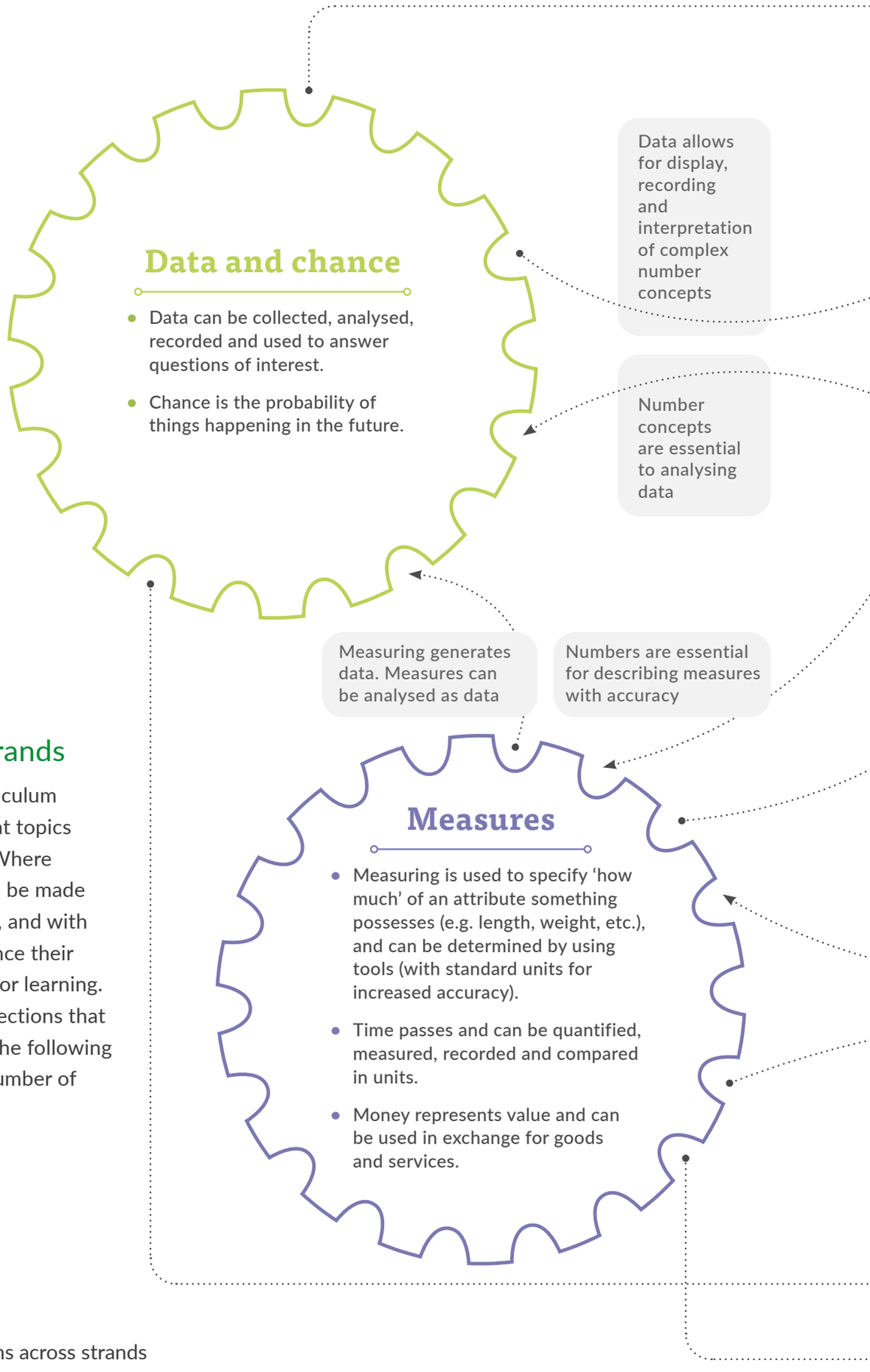


Figure 7: Examples of connections across strands

Data analysis can be used to organise information to uncover patterns and deduce relationships

Number

- Numbers can be used to quantify, label or tell the order of something.
- Counting allows us to quantify the amount of something in numbers.
- Ten is the base unit of our number system. It is the foundation for how we record, represent and calculate numbers.
- Number operations are used to calculate solutions accurately and efficiently.
- Fractions and decimals quantify parts of a whole.

Algebra represents numbers and relationships between different numbers

Algebra can be used to identify patterns, rules and relationships in number

Algebra

- Patterns and rules are key to establishing order and can be useful to make predictions and generalisations.
- Relationships can be conveyed as expressions and equations, which can then be used to find unknown information and solutions to problems.

Measures are a useful context for learning and understanding number

Numbers and number patterns can be represented geometrically

Shape and space have quantifiable attributes that can be described and measured using number

Algebra can be used to describe, map, identify and extend geometric patterns

Shape and space

- Objects, including our bodies, can be moved around for different purposes. Distance, direction and the location of objects in space can be analysed in mathematical terms using grids, maps and co-ordinates.
- Shapes can be composed and decomposed; and have attributes, which can be used for classification and analysis.
- Shapes can be transformed in a range of ways for a range of purposes.

The context of Shape and space is important for understanding and applying measures

Measuring quantifies some attributes of shape and space

Geometric representations can be useful in graphing and interpreting data

Equivalence can be determined and applied by understanding and examining relationships

5. Learning Outcomes

Learning Outcomes are used to describe the expected mathematical learning and development for all learners at the end of a two-year stage, when due account is taken of individual abilities and varying circumstances. Learning Outcomes articulate big mathematical ideas across different stages, and encompass the knowledge, skills and dispositions that children develop with the *Primary Mathematics Curriculum*. Some strand units have Learning Outcomes across all stages, some do not. These Learning Outcomes reflect the mathematical learning that is most appropriate for each stage.

Reflecting the principles and pedagogical approaches in the *Primary Curriculum Framework*, the 'stem' **'Through appropriately playful and engaging learning experiences'** is used to introduce Learning Outcomes across all stages. A playful and engaging approach to learning and teaching serves to present Mathematics as an open and accessible learning space, while also encouraging children to appreciate the beauty, challenge and power of Mathematics. This Learning Outcome stem emphasises a learning environment that motivates children to develop their mathematical proficiency and provides for rich learning experiences that reflect relevant pedagogical approaches as outlined in chapter 6, 'The *Primary Mathematics Curriculum* in Practice'.

The curriculum recognises that children learn and teachers teach in a variety of contexts. Therefore, the learning and teaching journey is varied and different across contexts. A Learning Outcomes approach recognises that teachers are best placed to determine the learning needs and strengths of the children in their class. They make decisions about what, and how, to teach and assess using appropriate pedagogical approaches and tools. Learning Outcomes, when shared with children, can support them to have clear expectations and to be active agents in their own learning.

A range of tools can be found in the Primary Mathematics Toolkit to support teachers in working towards Learning Outcomes across each stage.

	Learning Outcomes for Algebra Strand			
	Stage 1: Junior and senior infants	**Stage 2:** First and second classes	**Stage 3:** Third and fourth classes	**Stage 4:** Fifth and sixth classes
	Through appropriately playful and engaging learning experiences, children should be able to			
Patterns, rules and relationships	explore, extend and create patterns and sequences.	identify and express relationships in patterns, including growing or shrinking shape patterns and number sequences.	identify rules that describe the structure of a pattern and use these rules to make predictions. represent the relationships between quantities.	identify, explain and apply generalisations, including properties of operations, mathematical models and patterns. represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.
Expressions and equations		interpret the meaning of symbols or pictures in number sentences.	represent and express problems with known and unknown values in different ways to include the use of appropriate letter-symbols or words.	articulate, represent and solve mathematical situations through the use of expressions and equations that include letter-symbols.

Learning Outcomes for Data and Chance Strand				
	Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
	Through appropriately playful and engaging learning experiences, children should be able to			
Data	explore, interpret and explain data in a variety of ways for a range of purposes.	pose questions of interest, record and use data as evidence to answer those questions and communicate the findings.	pose questions of interest and collect, display and critically analyse data in a range of ways for a range of purposes and communicate the findings.	pose questions, collect, compare, summarise and represent data selectively to answer those questions. critically analyse and evaluate findings; and communicate inferences, conclusions and implications from the findings.
Chance			describe and test predictability and (un) certainty in events.	use probability to make informed decisions and predictions. represent and express probability in different forms.

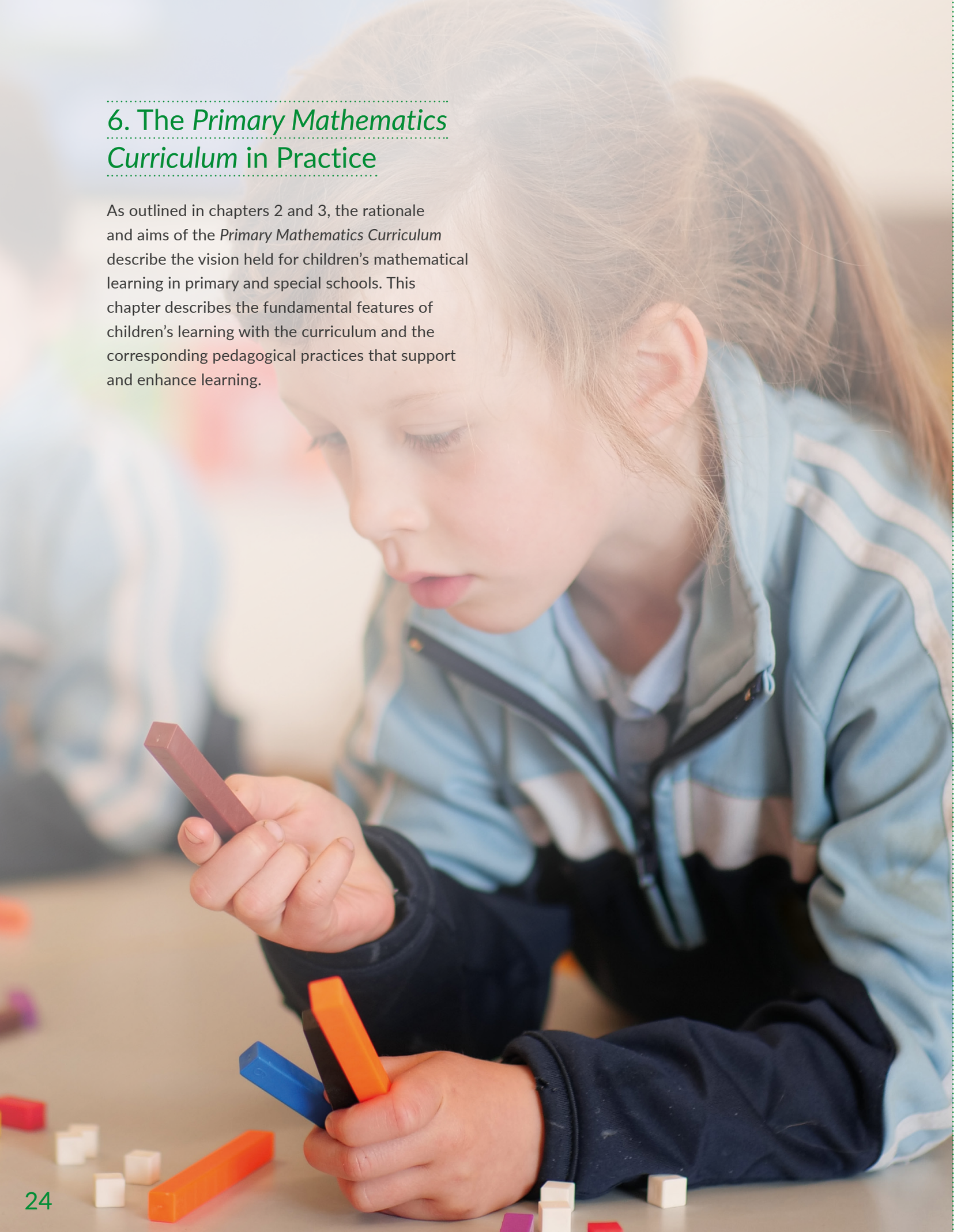
	Learning Outcomes for Measures Strand			
	Stage 1: Junior and senior infants	**Stage 2:** First and second classes	**Stage 3:** Third and fourth classes	**Stage 4:** Fifth and sixth classes
	Through appropriately playful and engaging learning experiences, children should be able to			
Measuring	demonstrate an awareness that attributes such as length, weight, capacity and area can be measured and compared.	compare, approximate and measure length, weight, capacity and area using appropriate instruments and record using appropriate units of measurement.	compare, estimate and measure length, weight, capacity, area and volume using appropriate instruments and record and communicate appropriately. identify the relationship between equivalent units of measurement, and rename measures using equivalent units.	determine and calculate units of measurement in fractional and/or decimal form to solve practical problems. find, interpret and deduce measures experimentally with increasing precision.
Time	develop a sense of time and its uses.	understand how time is measured, expressed and represented. explore equivalent expressions of time.	compare, approximate and measure time using appropriate units of measurement. identify the relationship between different units and representations of time.	solve and pose practical tasks and problems involving the interpretation and calculation of time.
Money	develop an awareness of money and its uses.	recognise the value of money and use euro and cent in a range of meaningful contexts.	transfer knowledge of the base ten system in number to monetary contexts and use for purposes of calculation.	solve and pose practical tasks to investigate and make informed judgements about transactions and financial plans.

	Learning Outcomes for Number Strand			
	Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
	Through appropriately playful and engaging learning experiences, children should be able to			
Uses of number	develop an awareness that numbers have a variety of uses.			
Numeration and counting	develop an awareness that the purpose of counting is to quantify. use a range of counting strategies for a range of purposes.	demonstrate proficiency in using and applying different counting strategies.		
Place value and base ten	develop a sense of ten as the foundation for place value and counting.	understand that digits have different values depending on their place or position in a number. use estimation to quickly determine number values and number calculations.	explore equivalent numerical expressions of numbers using the base ten system.	investigate how decimals and percentages (and fractions) can be compared, ordered and expressed in related terms.
Sets and operations	recognise and understand what happens when quantities (sets) are partitioned and combined.	select, make use of and represent a range of addition and subtraction strategies.	understand and apply flexibly the four operations; and the relationships between operations.	build upon, select and make use of a range of operation strategies.
Fractions	develop an awareness of part-whole relationships using a variety of models (area, length and set).	recognise and name fractions according to their part-whole relationships. explore the concept of equivalence in terms of simple fractions.	compare and express in equivalent terms; and order fractions. calculate the fraction of quantities and express in multiple ways.	explore (model, compare and convert) the relationships between fractions, decimals and percentages. investigate proportionality and ratios of quantities (sets).

	Learning Outcomes for Shape and Space Strand			
	Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
	Through appropriately playful and engaging learning experiences, children should be able to			
Spatial awareness and location	develop a sense of spatial awareness in relation to their bodies and the immediate environment. describe the spatial features of objects and their relative position in space.	use spatial knowledge for the purposes of orientation and navigation. visualise and model location using symbolic co-ordinates.	describe, interpret and record directional instructions and location. compare and classify angles, recognising them as a property of a shape and as a description of a turn.	describe location on the full co-ordinate plane. interpret scale maps and create simple scale drawings.
Shape	explore and recognise properties of 3-D and 2-D shapes.	examine, categorise and model 3-D and 2-D shapes.	investigate and analyse the properties of 3-D and 2-D shapes and identify classes of shapes based on these properties. represent shapes with drawings and models, and calculate dimensions of shapes.	construct 3-D and 2-D models or structures given defined measurements and/or specific conditions. investigate and construct angles in the context of shape; and solve angle-related problems.
Transformation	explore the effects of shape movements.	understand that shapes and line segments can be reflected, rotated and translated.	model and explain the effects of transformations on shapes and line segments.	perform and devise a range of steps involving transformations. analyse and show how shapes are enlarged on scaled diagrams.

6. The *Primary Mathematics Curriculum* in Practice

As outlined in chapters 2 and 3, the rationale and aims of the *Primary Mathematics Curriculum* describe the vision held for children's mathematical learning in primary and special schools. This chapter describes the fundamental features of children's learning with the curriculum and the corresponding pedagogical practices that support and enhance learning.



6a. Learning primary mathematics

A mathematics-rich learning environment provides an important context for children's learning experiences with Mathematics. In providing for playful and engaging mathematical learning experiences, it is essential to offer opportunities for children to:

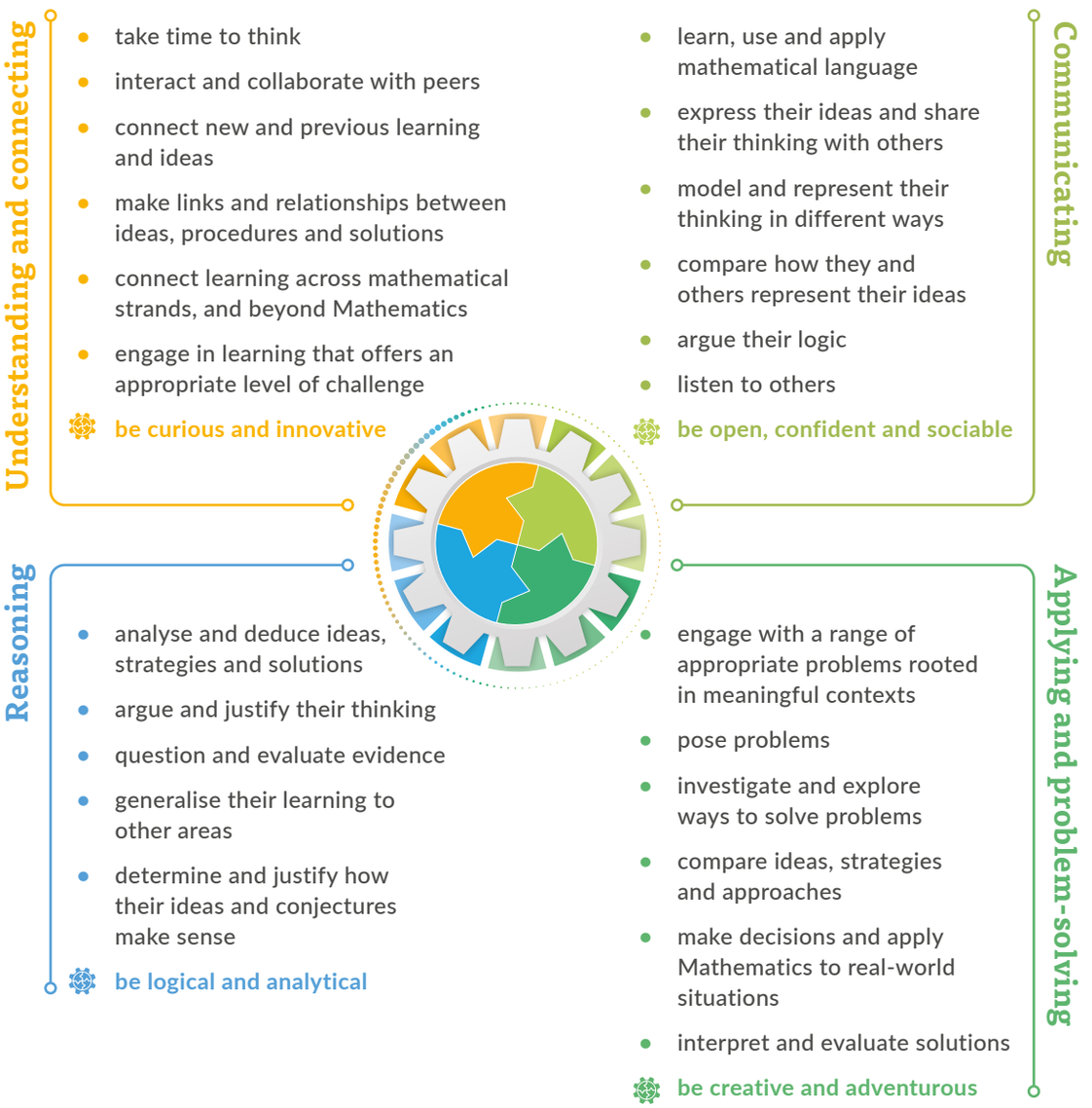


Figure 8: Mathematical learning opportunities

Technology and mathematical tools can be useful to support children's mathematical thinking and conceptual understanding. These can also reduce procedural load and support children to represent complex ideas.

6b. Teaching primary mathematics

'How' children learn is as important as 'what' children learn. The following five pedagogical practices are rooted in contemporary research. These practices are acknowledged as essential to the provision of quality mathematical learning experiences. They foster an inclusive learning environment and culture where children engage in rich and meaningful learning processes, as described in section 6a. Moreover, these pedagogical practices allow for children to learn and develop at a pace and level of challenge that is individual to their needs and interests whilst developing their confidence and proficiency in Mathematics. As such, these practices should permeate teachers' everyday decision-making about learning, teaching and assessment of Mathematics.

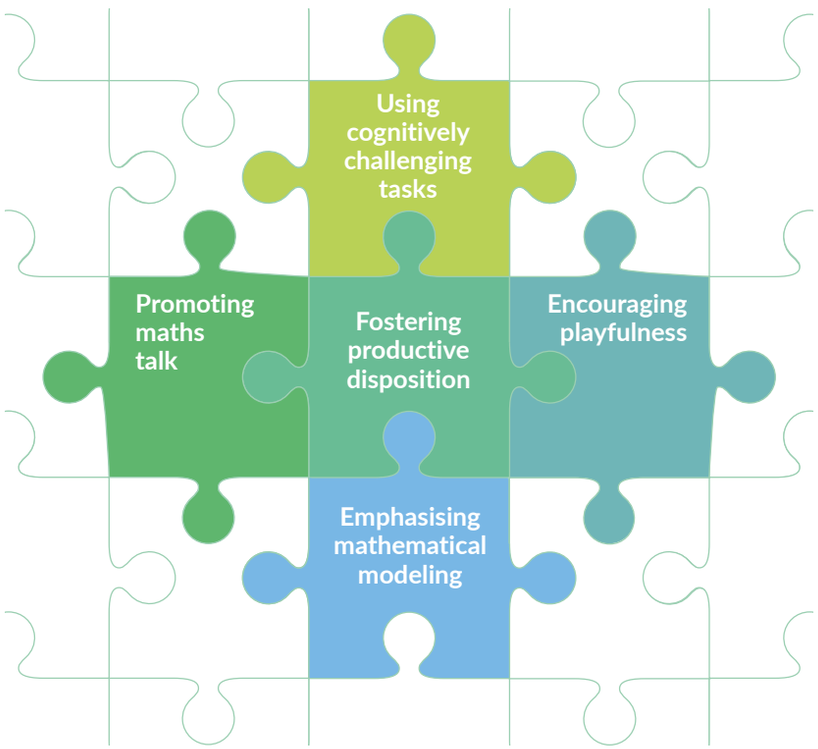
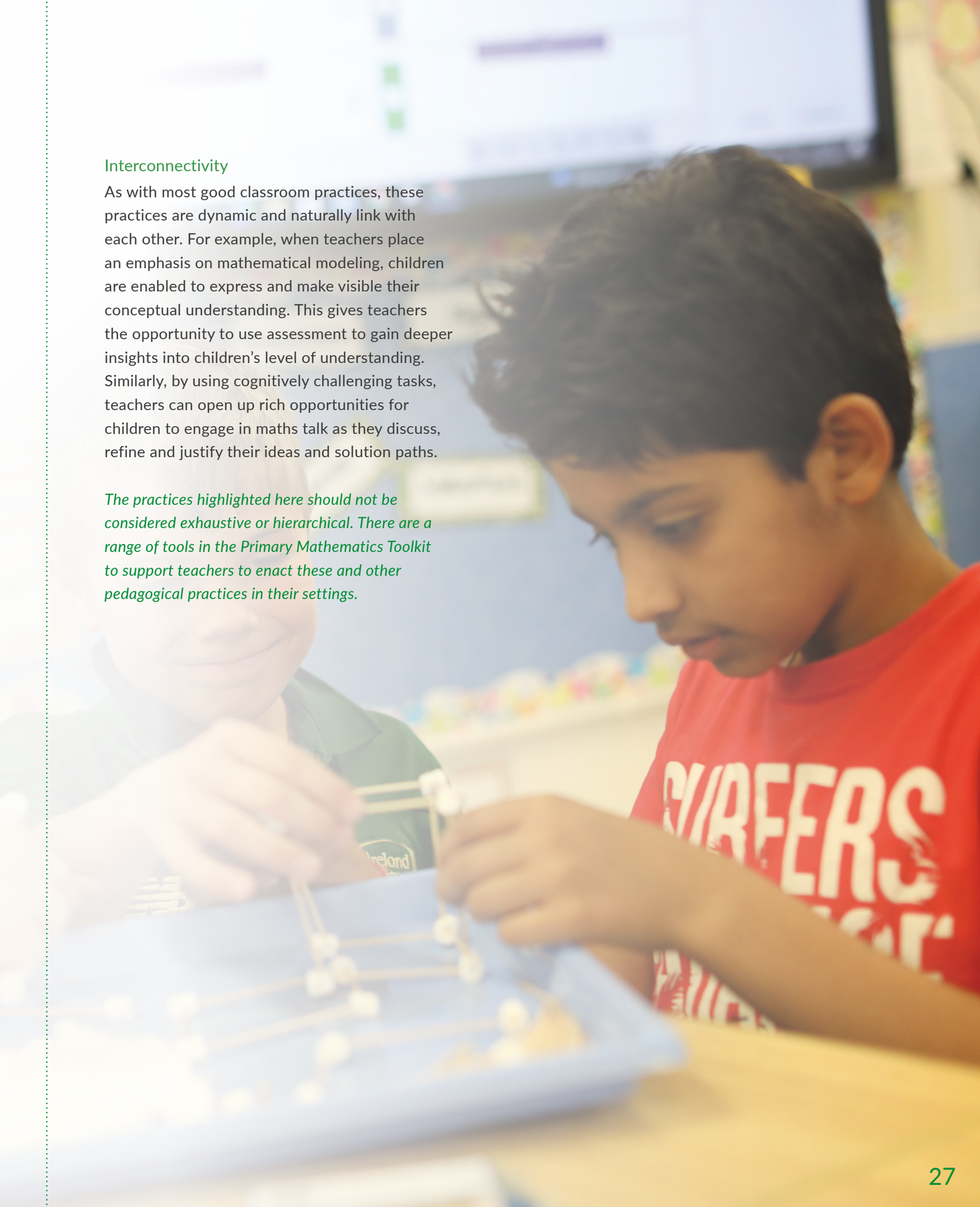


Figure 9: Five key pedagogical practices for the classroom

Interconnectivity

As with most good classroom practices, these practices are dynamic and naturally link with each other. For example, when teachers place an emphasis on mathematical modeling, children are enabled to express and make visible their conceptual understanding. This gives teachers the opportunity to use assessment to gain deeper insights into children's level of understanding. Similarly, by using cognitively challenging tasks, teachers can open up rich opportunities for children to engage in maths talk as they discuss, refine and justify their ideas and solution paths.

The practices highlighted here should not be considered exhaustive or hierarchical. There are a range of tools in the Primary Mathematics Toolkit to support teachers to enact these and other pedagogical practices in their settings.



Fostering productive disposition

Dispositions are not static and can be nurtured or changed over time. The multiple ways in which children engage with Mathematics, how they perceive Mathematics, and the rich contexts in which Mathematics is meaningfully presented to them, are what help form and shape their dispositions towards Mathematics. In addition, attitudes to Mathematics and values, both at home and in the classroom, also have a strong impact on the development of the child's productive disposition for Mathematics. A classroom which emphasises the rich, useful and meaningful nature of Mathematics has a very positive effect on the child's disposition to learning.

Teachers can help foster children's productive disposition by:

- demonstrating enthusiasm for Mathematics themselves
- providing rich and meaningful contexts for learning
- celebrating effort and success
- valuing the process as well as the product of learning
- normalising struggle and mistakes as part of the learning process
- giving children opportunities to interact and work collaboratively with their peers
- facilitating children to find patterns and make connections
- encouraging children to take risks and persevere
- engaging children in meaningful self-assessment and reflection.

Fostering a productive disposition to Mathematics is a shared responsibility

Families, teachers and other significant adults in the child's life all play a role in presenting a positive view of Mathematics. When children see the application of Mathematics in situations they care about and which are relevant to their lives, it is more likely that they will engage with Mathematics in a meaningful way. Teachers can promote the development of productive disposition in the home by suggesting fun and meaningful mathematical activities and games that can be used with children.

Many everyday activities that children enjoy offer opportunities for a rich engagement with Mathematics. There are supports available in the Primary Mathematics Toolkit for families to promote positive and purposeful engagement with Mathematics in the home environment.

Encouraging playfulness with Mathematics

Playful learning is appropriate for all children across all stages of primary and special education. Children's learning experience in Mathematics, as in all other areas of learning, is characterised by play and playfulness. Throughout childhood, play is of value in and of itself and children have both a right and a desire to play. Before mathematical language and concepts are formally introduced to children, they typically engage in a range of mathematical processes. These include testing, discovering, revising, extending, combining and transforming.

Play is an opportunity for teachers to engage with children in purposeful and sensitive ways. By infusing playfulness in children's learning experiences and the interactions between the teacher and child, children are encouraged to develop a productive disposition towards Mathematics, and to remain interested and engaged in the process of learning.

Mathematical learning can be greatly enhanced in a play environment that is interactive, engaging, inclusive and supportive; and that provides opportunities for exploration, investigation, challenge, creativity, choice and independence. Play provides a context for mathematical thinking and the development of mathematical language and concepts, with clear potential for promoting maths talk. Through play, children can be supported to engage in increasingly sophisticated and cognitively challenging activities and make choices about their learning. In doing so, their mathematical concepts are strengthened and extended; the tools, approaches and strategies they employ are refined; and the opportunities for children to make connections and share ideas are increased.

Teachers can help encourage playfulness with Mathematics by:

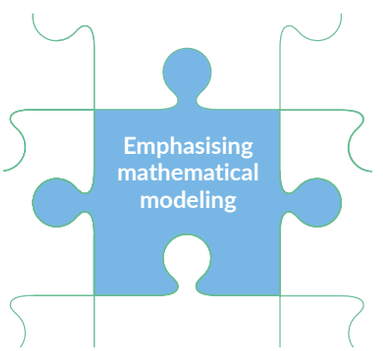
- being playful in their own dispositions and interactions with children
- tapping into children's interests and curiosities
- integrating mathematical learning with playful activities throughout the day
- signalling when children encounter Mathematics in spontaneous play and exploration
- introducing and reinforcing mathematical language as it arises through play
- encouraging multiple means of expression and representation
- providing opportunities for children to explore and experiment with mathematical ideas
- allowing a safe space for spontaneity, creativity and imaginative play with Mathematics
- providing access to a wide range of resources, visual supports and technologies.

Emphasising mathematical modeling

Mathematical modeling involves children using Mathematics to describe a problem-context and determine meaningful solutions to the problem. Children form models through a process of testing, revising and expressing their interpretation of different mathematical ideas, experiences, problems and situations, typically posed to them as questions or challenges. Children naturally generate their own informal mathematical models in a way that is context-specific and makes sense to them. As children's knowledge, understanding and experience grows, they may develop more formal, sophisticated and efficient models which they can use to share, connect and communicate their ideas with others. They may also transfer these models to a range of different contexts in a way that is meaningful to themselves and others. In forming models, children might use physical actions, spoken words, objects, images (e.g., graphs, diagrams and pictures), symbols or written words. While accuracy is valued, mathematical modeling places more importance on exploration, sense-making, conceptual understanding and flexibility in thinking.

Teachers can help emphasise mathematical modeling by:

- providing opportunities for sense-making
- allowing freedom and autonomy for children to develop and express their own models and solution pathways
- using model-eliciting activities, questions, prompts and feedback to provoke situations for modeling
- encouraging individuality, choice and independence
- facilitating children to build, test and apply mathematical models
- challenging children to test and refine their models through collaboration
- celebrating diversity and creativity in working with mathematical models
- supporting children to generalise their models for a range of different contexts and purposes.



Using cognitively challenging tasks

Cognitively challenging tasks are rich, higher-order learning opportunities that should appropriately stretch and challenge children's conceptual understanding as they encounter significant mathematical ideas and situations. Sometimes referred to as low-threshold high-ceiling tasks, these tasks should provide all children with the opportunity to access Mathematics, while offering the potential for deeper engagement. Simple, considered and well-pitched tasks can present a rich medium through which children can engage meaningfully in mathematical content and processes. They also offer opportunities for teachers to incorporate other key pedagogical practices, such as maths talk and mathematical modeling.

When used effectively, children perceive and experience these tasks as having few prescribed or memorised rules or methods; as well as an opportunity to freely explore different ways of solving problems. It is through exploring these tasks and grappling with problems and solutions in meaningful ways that children deepen their understanding of Mathematics. Through their efforts to engage with cognitively challenging tasks, children develop persistence and resilience which are essential to a productive disposition for Mathematics.

Teachers can help promote the use of cognitively challenging tasks by:

- selecting, designing or modifying tasks to appropriately stretch and deepen children's understanding
- providing opportunities for deep and sustained engagement with mathematical content and processes through the use of tasks
- allowing children to grapple with ideas and problems freely and to explore problems with multiple correct solution pathways
- encouraging different ways of solving problems
- assisting children to make connections with prior and new learning
- encouraging children to express and communicate their ideas frequently and openly
- holding high expectations for what children are capable of understanding, doing and communicating
- providing opportunities for children to collectively share and evaluate their experiences from working with tasks
- celebrating individual and collaborative effort and success in grappling with challenging tasks.

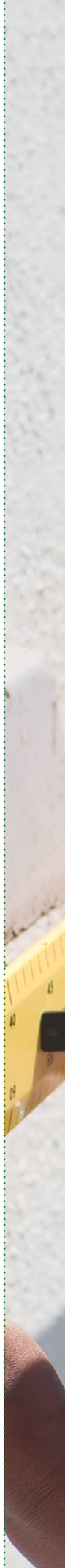
Promoting maths talk

Maths talk is a collaborative process where children's thinking, strategies and ideas are expressed, shared and/or exchanged. This allows children to reflect on their own understanding; define, present and justify their ideas; make sense of and critique their own ideas and those of others; and develop their ability to express and articulate their thinking. Through maths talk, children can engage in rich mathematical processes which deepen their understanding of Mathematics. For example, by presenting, arguing or justifying their mathematical ideas, they can refine, consolidate or extend their existing knowledge.

Maths talk equips children with tools to make their thinking visible. These tools include words, sign language, body language and gestures, symbols, diagrams, concrete manipulatives and technology. All children are mathematical language learners, regardless of their language proficiency, and all can engage in maths talk. Opportunities should be provided for children to communicate in ways that are meaningful for them. The learning environment should be flexible in terms of recognising and catering for multiple forms of expression and multiple means of engagement. The physical space should provide children with access to a variety of materials that stimulate and enable maths talk.

Teachers can help promote the use of maths talk by:

- providing a safe environment for children to share and exchange thinking and ideas
- encouraging active listening, respect and value for all contributions
- identifying and selecting appropriate situations and problems to promote maths talk
- re-casting everyday experiences using mathematical words and phrases
- prompting maths talk through strategic, skilful, open and thoughtful questioning
- providing suggestions for parents on how to promote and stimulate maths talk at home
- allowing waiting time and time for sustained interactions, collective sharing and reflection
- re-voicing children's mathematical ideas.



6c. Assessing primary mathematics

Assessment is an integral part of learning and teaching. It involves teachers and children working together to use information to inform and support learning and teaching. Teachers are committed, skilled and agentic professionals who make key decisions every day about mathematical learning and teaching. These decisions are informed and shaped by:

- knowledge of the child and their prior learning
- knowledge of the curriculum
- knowledge of pedagogy.

The child and their prior learning

Children engage with and process their learning in Mathematics in different ways depending on their age, ability, strengths, preferences and interests. Given the incremental nature of mathematical learning, having a solid knowledge of children's prior learning and understanding is fundamental. This knowledge informs the appropriate learning sequence and scaffolds that support children to build on and deepen their knowledge, concepts and skills. Prior learning can be assessed through any activity that offers children opportunities to express their understanding and reasoning. The more that is understood about where children are at in their learning journey and how they learn, the better their mathematical understanding and productive disposition can be nurtured, and the learning environment tailored to meet their needs.

The curriculum

As children work towards Learning Outcomes, and develop and deepen their mathematical proficiency, assessment provides useful insights and information about children's progress. This information can be observed, interpreted, and used by teachers or children in a responsive way to support ongoing decision-making about the next steps for the child's mathematical learning journey.

Pedagogy

By reflecting on the learning opportunities provided to children (as described in section 6a) and mathematics-related pedagogies enacted in the classroom (as described in section 6b), teachers can refine and adjust both the learning experiences and the learning environment. This serves to ensure that teachers are responding appropriately to children's learning. Professional conversations with colleagues, continuing professional development and accessing mathematical teaching resources and tools provide further supports for teachers.

Children as mathematical learners: Providing children with regular time to talk about their learning, reflect and determine their next steps contributes to their identity and confidence as mathematical learners. In addition to informing learning and teaching, when used effectively, assessment can also help children to become increasingly independent and motivated in learning Mathematics.

By equipping children with appropriate skills, and by sharing the focus of learning and agreeing criteria for successful learning with them, teachers can give children the tools they need to peer and self-assess, reflect on and take greater responsibility for their own mathematical learning.

Opportunities for assessing mathematical learning: The ability to recognise Mathematics in children's everyday activities and to extend the potential learning arising from these everyday activities is critical to planning for assessment in the classroom. Children's mathematical learning can be assessed along a continuum from 'intuitive' to 'planned interactions' to 'assessment events' as shown in Figure 10. The three types of assessment are complementary, and necessary, to gain a comprehensive picture of a child's progress and achievement.

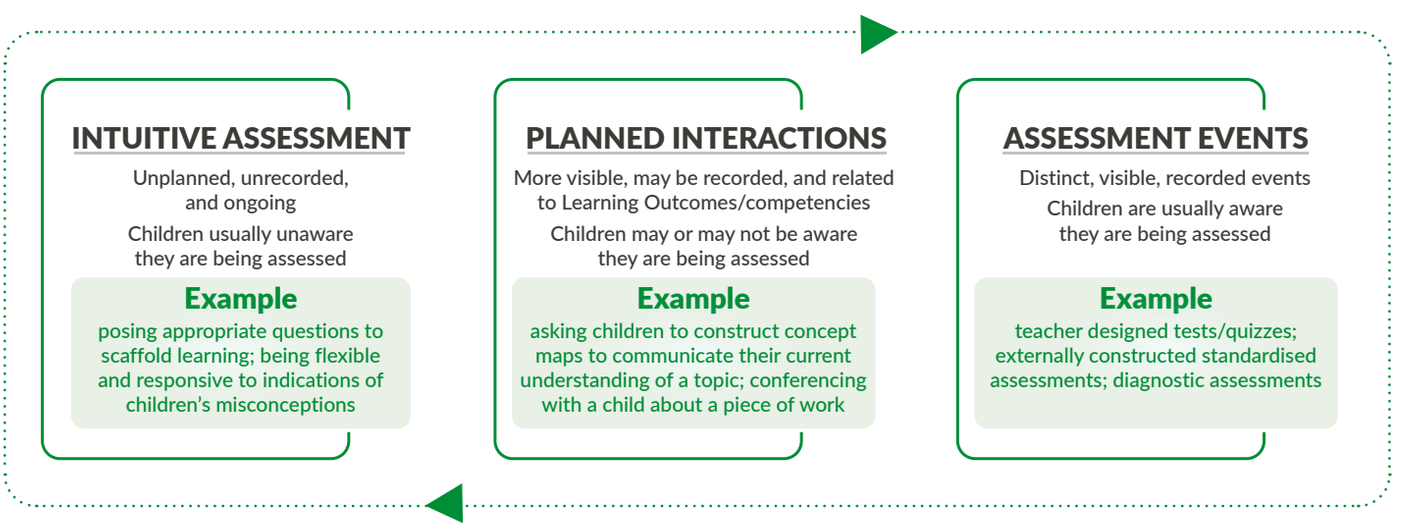
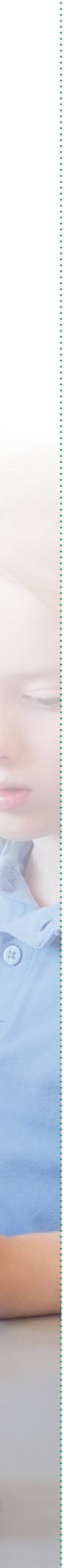


Figure 10: Assessment continuum

During the mathematical learning process, assessment is likely to occur incidentally, intuitively and in response to contingency moments i.e. unplanned and unexpected responses from the child. Providing an openness for such contingency moments, and capitalising where appropriate on insights gleaned from children's responses, questions, problems and tasks, can allow for very rich assessment data.

The Primary Mathematics Toolkit contains supports which may be useful for teachers in assessing children's mathematical learning.



Methods for assessing mathematical learning

As teachers continually assess children's mathematical knowledge, skills and dispositions, they are likely to use multiple assessment methods, in inter-changeable ways, to build a rich picture of children's mathematical learning. Below are some of the methods likely to be used.

Observations: Teachers are well placed to actively monitor and observe children as they engage in mathematical learning experiences and to respond to opportune learning situations as they arise organically.

Questioning: Effective, well-crafted and probing questions that are open, related to the focus of learning and accompanied by appropriate wait time encourage children to think deeply, develop their understanding and express their ideas and solutions.

Conferencing: Through teacher/child, peer/peer and group meetings, teachers can gain an insight into children's mathematical thinking, the nuances of their learning and their experience of learning.

Tasks: Tasks can be very useful to provide an insight into children's level of understanding and their preferred methods for learning Mathematics. Assessment tasks can be written, oral or practical.

Feedback: The provision of feedback, focused on the mathematical learning or task in hand, can help children identify and celebrate their progress and achievements, pinpoint challenges they experience, give direction for future work and decide what the appropriate next steps might be.

Portfolios: Portfolios can be assembled, digitally or otherwise, to compile evidence of children's mathematical learning and provide a source of self-reflection, feedback and assessment. Artefacts could include pictures, recordings and work samples, among others.

Summative tests: When used in conjunction with other forms of assessment, analyses from summative tests can provide an important source of detailed feedback on children's learning.

Peer and self-assessment methods: Teachers may also use strategies to support children to engage in peer and self-assessment and rely to a lesser degree on extrinsic affirmation and motivation. An open, respectful and collaborative classroom culture and learning environment is essential to support children to think about their own learning and that of others. Children working in pairs or small groups facilitates group reflection and feedback sharing. It may be useful to display children's work to allow for peer and self-assessment.

Peer and self-assessment can also be promoted by:

- allowing time and space for children to reflect on and discuss their mathematical learning
- making explicit to children what they are going to learn, how they might build on prior knowledge and possible connections with other areas of Mathematics familiar to them
- suggesting criteria for children to use in monitoring their progress
- promoting and celebrating respectful and open collaboration, reflection and sharing.

7. Outline of the Primary Mathematics Toolkit

The Primary Mathematics Toolkit provides practical support for teachers in building rich mathematical learning experiences for children.

The components of the toolkit are mathematical concepts, progression continua, support materials and examples of children's learning.

Mathematical concepts

Mathematical concepts are considered key ideas that underpin each Learning Outcome. These key ideas may provide useful entry and reference points in relation to planning, teaching and assessment and may serve to remind teachers of key mathematical knowledge at each stage. The mathematical concepts are situated in the Primary Mathematics Toolkit. They are presented according to stages 1 to 4 and link with the corresponding Learning Outcomes. Children will develop their understanding of these corresponding mathematical concepts through engaging with the mathematical processes, as outlined in the elements.

Progression continua

In working towards Learning Outcomes, teachers seek to engage children in a range of appropriately playful and engaging learning experiences. These learning experiences should be responsive to the needs, interests and abilities of every child. The progression continua outline a sample learning trajectory of Mathematics at primary level. They suggest a series of learning experiences, which children might engage with as they develop and deepen their mathematical knowledge, skills and dispositions.

Classrooms are complex and children come to school with different experiences and learn in diverse ways. They also learn and develop at different rates. Therefore, while the progression continua suggest a sample learning journey in Mathematics at primary level, they are not intended to be prescriptive or exhaustive. Indeed, children rarely learn in a linear or typical way. Some children may move forwards and backwards across the continua for different strands or elements of mathematical learning. Others may work within one progression step or across a small number of steps for the duration of their primary years. Teachers should exercise professional judgement when making decisions as to the learning experiences which are most appropriate for the children in their classroom.

There are fifteen progression continua tables, one for each of the strand units. Each continuum describes the learning journey across eleven progression milestones (a-k) in terms of mathematical content and processes.

Support materials

The Primary Mathematics Toolkit includes a range of support materials for teachers to use with the *Primary Mathematics Curriculum*. The support materials include important resources and reference material to support teachers to enact the curriculum in a meaningful way. Support materials include descriptions and examples of the pedagogical practices outlined in chapter 6, supports for fostering a rich learning environment, as well as supports for promoting links with home and community. These materials are developed and reviewed in response to research and the evolving needs of children, teachers and school communities.

Examples of children's mathematical learning

The Primary Mathematics Toolkit includes a range of examples of children's mathematical learning and development. These examples illustrate teacher-child interactions, cognitively challenging tasks, learning in integrated contexts and problem-based learning. In doing so, it is intended that these examples will exemplify playful and engaging learning experiences and demonstrate good pedagogical practices as children work towards Learning Outcomes.

8. Glossary

The glossary highlights key words and terminology used in the curriculum which may be new to teachers or which may require further explanation.

Argumentation	A dynamic process for discovering and understanding new mathematical ideas and presenting the rationale for same.
Child agency	Children are active in their own learning and can display their agency by taking the initiative in learning situations, by observing and becoming involved in ongoing events, or by initiating conversations with others.
Collaborative learning	Learning that takes place in social contexts and uses the resources of the environment.
Conceptual understanding	Understanding of mathematical concepts, operations and relations.
Conferencing	Dedicated time, space or meeting to elicit and gain a deeper insight into children's level of understanding.
Conjecture	An educated guess that is based on known and/or incomplete information.
Contingency moments	Unplanned or unexpected events or responses from children that occur during the learning process.
Co-ordinate plane	A two-dimensional plane divided into four quadrants.
Data	A collection of information or facts, such as numbers, words, measurements, observations or other descriptions.
Disposition	An enduring habit of mind and action. The tendency to respond to situations in characteristic ways.
Generalising	To make assertions, claims or justifications as to how children's understanding is applicable or transferrable to other circumstances.
Learning environment	Describes any space in which children learn or develop their understanding.
Low-threshold high-ceiling tasks	Tasks that provide accessible entry points for learning with the scope for exploration and challenge for all learners.

Mathematical modeling	Involves using Mathematics to conceptualise a problem or situation and determine meaningful solutions, and in doing so help children formalise their mathematical learning in a way that makes sense to them.
Mathematical proficiency	Consists of the five intertwined and interrelated strands of conceptual understanding, procedural fluency, adaptive reasoning, strategic confidence, and productive disposition.
Maths talk	A collaborative process where children's thinking, strategies and ideas are discussed, shared and exchanged
Peer assessment	Involves children looking at each other's work in a reflective way.
Productive disposition	The inclination to see Mathematics as something worthwhile, useful and doable.
Re-voicing	The teacher repeats some or all of what the child has said and then asks the child to clarify whether or not this may be correct.
Rotation	Turning around a centre point.
Scaffolding	Describes the process by which teachers support and guide children's learning, by building on their current knowledge and experience.
Self-assessment	Involves children looking at their own work in a reflective way.
Summative assessment	Assessment is summative when it is used to evaluate children's learning at the end of the instructional process or of a period of learning.
Symbolic co-ordinates	Used to describe the position or location of a point or object.
Translation	A shape or line is translated when it is moved a certain distance from its original position (without turning).

References

Delaney, S. (2020). *Number in the Senior Primary Classes: Commissioned Research Paper for National Council for Curriculum and Assessment*. Accessed at https://ncca.ie/media/4622/primary_maths_research_number_seniorclasses.pdf

Dooley, T. (2019). *Learning and Teaching Primary Mathematics: An Addendum to NCCA Research Reports 17 and 18*. Accessed at https://ncca.ie/media/4087/primary_maths_research_addendum_2019.pdf

Dooley, T., Dunphy, E., & Shiel, G. (2014). *Mathematics in Early Childhood and Primary Education. Research Report 18*. Dublin: National Council for Curriculum and Assessment. Accessed at https://ncca.ie/media/2147/ncca_research_report_18.pdf

Dunphy, E. Dooley, T. & Shiel, G. (2014). *Mathematics in Early Childhood and Primary Education. Research Report 17*. Dublin: National Council for Curriculum and Assessment. Accessed at https://ncca.ie/media/1494/maths_in_ecp_education_theories_progression_researchreport_17.pdf

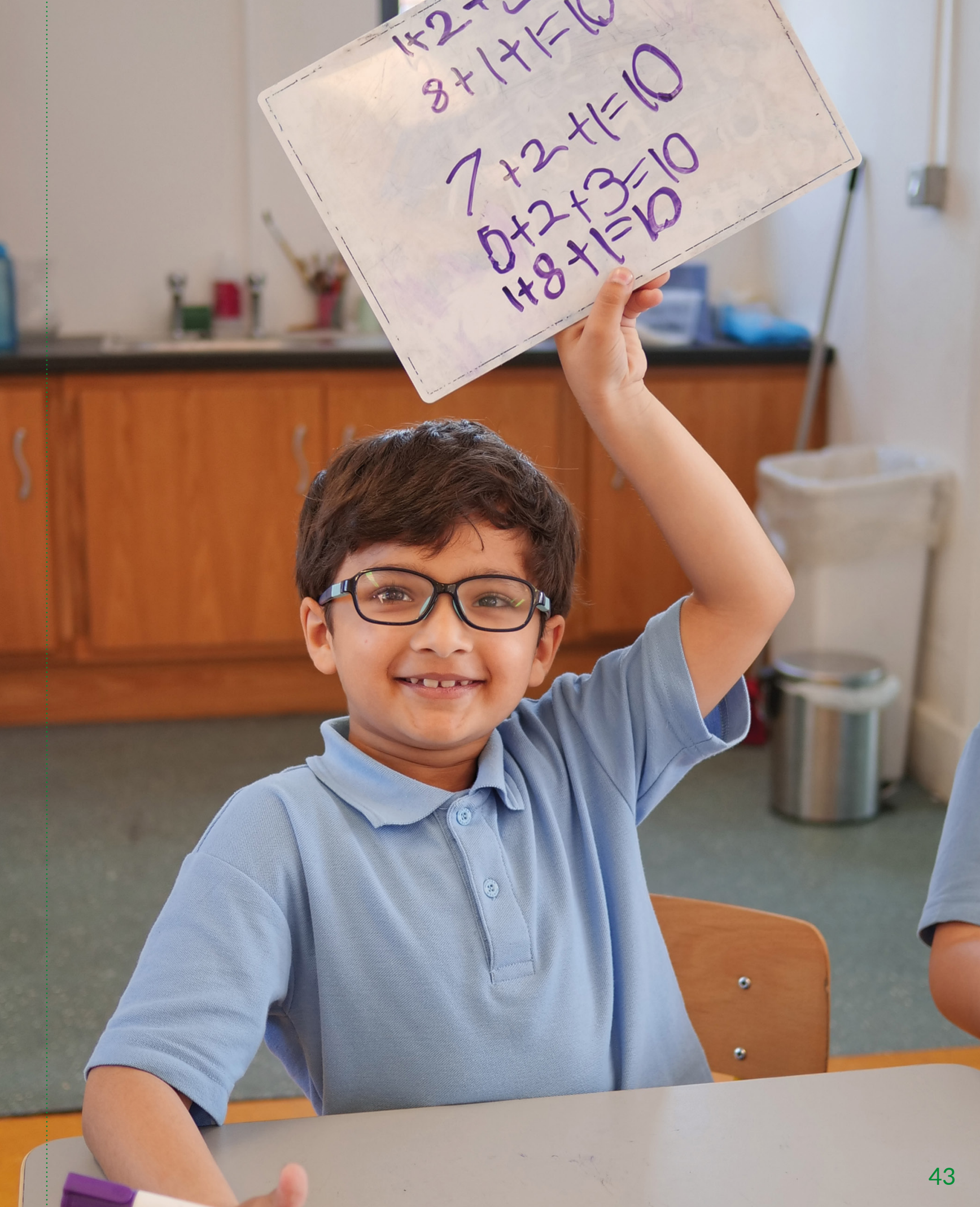
Leavy, A. (2020). *Data and Chance in the Senior Primary Classes: Commissioned Research Paper for National Council for Curriculum and Assessment*. Accessed at https://ncca.ie/media/4620/primary_maths_research_data_and_chance_seniorclasses.pdf

National Council for Curriculum and Assessment (2016). *Background Paper and Brief for the development of a new Primary Mathematics Curriculum*. Dublin: National Council for Curriculum and Assessment. Accessed at https://ncca.ie/media/1341/maths_background_paper_131016_tc.pdf

Nic Mhuirí, S. (2020a). *Measures in the senior primary classes: Commissioned research paper for National Council for Curriculum and Assessment*. Accessed at https://ncca.ie/media/4626/primary_maths_research_measures_seniorclasses.pdf

Nic Mhuirí, S. (2020b). *Shape and Space in the Senior Primary Classes: Commissioned Research Paper for National Council for Curriculum and Assessment*. Accessed at https://ncca.ie/media/4624/primary_maths_research_shape_and_space_seniorclasses.pdf

Twohill, A. (2020). *Algebra in the Senior Primary Classes: Commissioned Research Paper for National Council for Curriculum and Assessment*. Accessed at: https://ncca.ie/media/4619/primary_maths_research_algebra_seniorclasses.pdf


8+1+1=10
7+2+1=10
0+2+3=10
1+8+1=10

Notes

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Department of Education